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BACKSCATTERING OF ELECTROMAGNETIC WAVES FROM A SURFACE COMPOSED OF TWO TYPES OF SURFACE ROUGHNESS

by

Richard A. Hevenor

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13. ABSTRACT

This report presents a vector theory for the backscattering of electromagnetic waves from a random, rough surface. The basic technique used is that employed by Dr. Adrian K. Fung in an earlier work. The surfaces used in the report are those that are generated by a stationary gaussian random process as opposed to surfaces generated by a random array of objects. The former type of surface is assumed to simulate many of the vegetation-free sections of the earth. It is important to understand the basic characteristics of scattering from such surfaces for the purpose of aiding military geographic analysis by radar.

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Project 4A062112A854

October 1971

Distributed by

The Commanding Officer
U. S. Army Engineer Topographic Laboratories

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Geographic Information Systems Branch
Geographic Sciences Division

SUMMARY

This report presents a vector theory for the backscattering of electromagnetic waves from a random, rough surface. The basic technique used is that employed by Dr. Adrian K. Fung in an earlier work. The surfaces used in the report are those that are generated by a stationary gaussian random process as opposed to surfaces generated by a random array of objects. The former type of surface is assumed to simulate many of the vegetation-free sections of the earth. It is important to understand the basic characteristics of scattering from such surfaces for the purpose of aiding military geographic analysis by radar.

FOREWORD

The authority for performing the work described in this report is contained in Project 4A062112A854, Military Geographic Analysis.

The theory described herein is the result of in-house work and is based partially on the work of other investigators—in particular, Dr. Adrian K. Fung. The author wishes to thank Mr. Francis G. Capece and Mr. Regis J. Orsinger for aiding in checking the derivations and in writing the computer program. This task was performed under the supervision of Mr. Bernard B. Scheps, Chief, Geographic Information Systems Branch, and Dr. Kenneth R. Kothe, Chief, Geographic Sciences Division. The work was under the general direction of Mr. Gilbert G. Lorenz, Technical Director, U. S. Army Engineer Topographic Laboratories, U. S. Army Topographic Command.

CONTENTS

GLOSSARY OF SYMBOLS

σ^{o}	backscatter coefficient; ratio of differential radar cross section to differential surface area.
γ	backscatter coefficient; ratio of differential radar cross section to differential projected area.
θ	angle of incidence.
θ'	local angle of incidence.
λ	incident wavelength.
k	wavenumber in the propagating medium above the surface.
k'	wavenumber in the propagating medium below the surface.
$\rho(x, y)$	function describing the total composite surface.
$s(x, y)$	function describing the slightly rough surface.
$Z(x, y)$	function describing the large undulations.
$\vec{n}_1$	unit vector in the direction of the incident wave.
$\vec{r}$	range vector from the origin of the (x, y, z) coordinate system to the surface point.
$\vec{n}$	unit vector normal to the total surface, $\rho(x, y)$.
P	the point where the scattered field is computed.
$\vec{R}_1$	unit vector in the direction of P from the origin of the coordinate system.
$\vec{n}_2$	unit vector in the direction of P from a surface point.
ω	angular frequency of the wave.
μ	permeability of the medium below the surface.
ϵ	permittivity of the medium below the surface.
η	intrinsic impedance of the medium above the surface.
$\vec{E}$	total electric field on the surface.
$\vec{H}$	total magnetic field on the surface.
$\vec{E}_s$	scattered field at point P and in the direction defined by the unit vector, $\vec{n}_2$.
R	the distance from the origin to the point P.
$\vec{E}_i$	locally incident field.
$R_\perp$	Fresnel reflection coefficient for a local horizontally polarized plane wave.

GLOSSARY OF SYMBOLS (Cont'd)

$R_{\|\|}$	Fresnel reflection coefficient for a local vertically polarized plane wave.
$S(k_x, k_y)$	Fourier transform of the slightly rough surface function $s(x, y)$.
$C(r)$	autocorrelation coefficient for the large undulations.
$C_1(r)$	autocorrelation coefficient for the slightly rough surface.

BACKSCATTERING OF ELECTROMAGNETIC WAVES FROM A SURFACE COMPOSED OF TWO TYPES OF SURFACE ROUGHNESS

I. INTRODUCTION

1. **Purpose.** The purpose of this report is to present a general vector theory for the backscattering of electromagnetic waves from random, rough surfaces. The surfaces under consideration will be those generated by a stationary, random process as opposed to those generated by a random array of objects.

2. **Background.** At the present time, the method of extracting military geographic information about terrain features from radar is almost solely dependent upon the qualitative analysis of radar imagery by a photointerpreter. Radar should be a quantitative tool in view of the controlled frequency, look angle, and self-contained illumination system. In order to improve upon the present process and to make it more quantitative, an understanding of the process of electromagnetic wave interaction with and scattering from natural surfaces must be obtained. A general solution to the problem of plane-wave scattering from an arbitrary rough surface is still lacking. However, recent advances in vector approximate solutions have yielded considerable insight into the scattering phenomena. Radar scattering theories are basically separated into two broad classes: those which deal with surfaces generated by stationary, random processes; and those which deal with surfaces generated by a random array of objects. This report will deal exclusively with stationary, random processes because they are, in general, more realistic of natural terrain than the random array of objects. The parameters that affect the scattering of an electromagnetic wave from a rough surface must be isolated and molded into a vector theory which will allow the calculation of average return power. The average return power can then be used to calculate the backscatter coefficients σ^o and γ . The backscatter coefficient σ^o can be defined as the ratio of differential radar cross section to differential surface area, while γ can be defined as the ratio of differential radar cross section to differential projected area. A simple relation exists between γ and σ^o which is:

$$\sigma^o = \gamma \cos\theta$$

where θ is the angle of incidence. The two backscatter coefficients σ^o and γ are functions only of terrain properties and are not functions of radar parameters. One of the most difficult problems associated with rough-surface scatter is the question of depolarization. If a plane wave is incident onto a rough surface with, say, horizontal polarization, it is possible to receive not only a horizontal component but also a vertical component. This is due to the fact that the rough surface will scatter some of the original incident field into a different polarization, i.e., depolarization takes place. This very brief initial statement on depolarization is sufficient to permit a look into the fundamental parameters that affect the scattering of an electromagnetic wave from a rough surface. Five basic parameters listed and discussed by Cosgriff, Peake, and Taylor[1] are:

a. Surface Roughness

b. Angle of Incidence

c. Polarization

d. Complex Dielectric Constant

e. Frequency

The surface-roughness parameters would include such things as the probability distribution of the surface heights about a mean plane and the surface autocorrelation function. The surface-roughness parameters are the predominant factors in determining the scattered field. A qualitative understanding of surface roughness can be obtained through the Rayleigh criterion which relates the "roughness" of a surface to wavelength and angle of incidence. To this date, no thorough quantitative measure of surface roughness has been developed which may explain the fact that a general, exact solution to the problem of wave scattering from rough surfaces is still lacking.

The angle of incidence refers to the angle between the direction of the incident wave and the vertical. The plane of incidence is then the plane containing the wave-propagation direction and the vertical. In Fig. 1, the arrow located in the xz-plane represents the direction of the incident wave with angle of incidence, θ, and the plane of incidence is the xz-plane. At the far ranges, the angle of incidence will be large which usually results in difficult problems with shadowing. At near ranges, the angle of incidence will be small. The angle of incidence at a point on a surface can also be defined as the angle between the direction of the incident wave and the normal to the surface. In this case, the angle of incidence is usually referred to as the local angle of incidence.

The polarization of an electromagnetic wave is defined by the direction that the electric field vector takes. If this direction is constant in time, then, the wave is said to be linearly polarized. This report will consider two types of linear polarization in particular;

[1] Cosgriff, Peake, Taylor. *Terrain Scattering Properties for Sensor System Design (Terrain Handbook II)*, The Ohio State University, Engineering Experiment Station Bulletin No. 181.

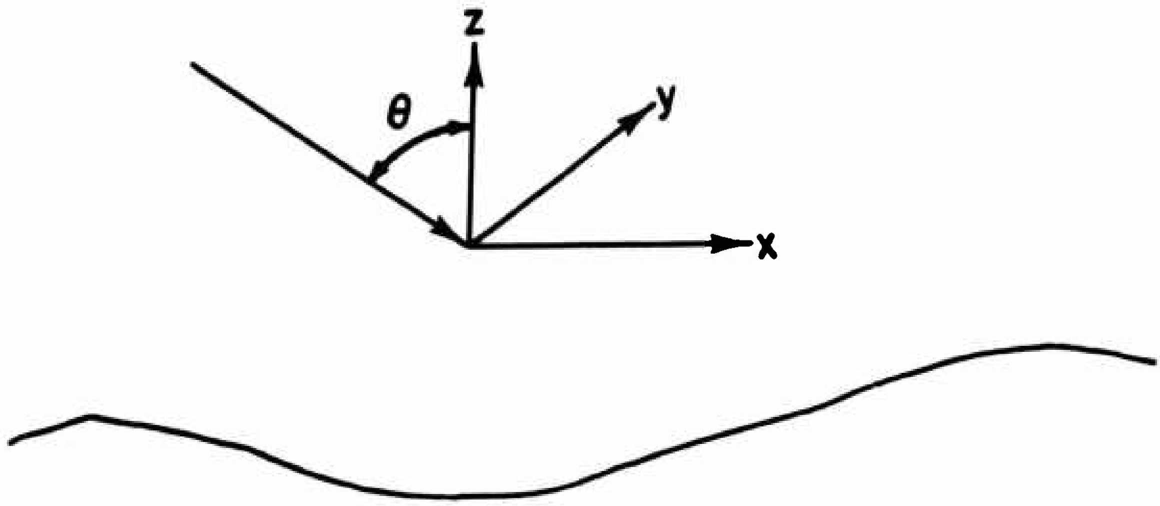

Fig. 1. The arrow located in the xz-plane represents the direction of the incident wave with angle of incidence, θ, and the plane of incidence is the xz-plane.

horizontal and vertical. Horizontal polarization occurs when the electric field vector is perpendicular to the plane of incidence and the magnetic field vector lies in the plane of incidence. Vertical polarization occurs when the magnetic field vector is perpendicular to the plane of incidence and the electric field vector lies in the plane of incidence. Polarization effects come into being not only because of the direction of the incident field vector but because of the depolarization properties of the surface. The complex dielectric constant is a function of the surface electrical properties (permittivity and conductivity) and the frequency of the incident wave.

Two basic techniques have evolved for working the problem of electromagnetic wave scattering from a surface generated by a stationary random process. This type of surface is one in which the elevations or heights of the surface above a mean plane are distributed with a certain probability density distribution function. The distribution function usually assumed is gaussian in order to help simplify the calculations. The tangent-plane method of computing wave scattering problems consists of approximating the local fields at a point on the surface by the fields which would be present on a plane tangent to the surface at the desired point. The calculated surface fields are then placed in the Helmholtz integral for the scalar case or the Stratton-Chu integral for the vector case. The scattered field in any particular direction can then be computed. In order for the tangent-plane approximation of local surface fields to be correct, the radii of curvature of the surface at all points must be much greater than the incident wavelength. An exact geometrical relationship which gives the criterion for the tangent-plane method was developed by Brekhovskikh[2] and can be stated as follows:

$$4\pi r_c \cos\theta' >> \lambda$$

where r_c is the smaller of two radii of curvature of a surface at a point. The angle θ' is the local angle of incidence, and λ is the incident wavelength. Therefore, it can be easily seen that the tangent-plane method will allow only the use of smoothly undulating surfaces and that no roughness which is smaller than a wavelength can be tolerated.

The other basic method of solving the scattering problem is known as the small perturbation method. In this method, the Fourier series or Fourier transform is used in a perturbation series to solve for the unknown fields above and below the rough surface. The exact boundary conditions are used; and, theoretically, this method would yield a good solution to the scattered field from any surface if enough terms of the perturbation series could be computed. In actual practice, it becomes exceedingly difficult to take much more than the first term of the series, particularly if finite conduction on the surface is allowed. Since the perturbation series effectively represents the results of the surface perturbations about a flat plane, then only slightly rough surfaces can be tolerated with this method. A quantitative definition of a slightly rough surface could be as follows:

$$|k s(x, y)| < 1$$

where k is the wavenumber in the propagating medium above the surface and $s(x,y)$ is a surface function representing elevations from a mean plane to the surface.

The tangent plane method has been shown to yield good results for the average backscattered power with the like-polarized component, at least for small angles of incidence (0°-20°) when a gaussian autocorrelation function is used. When an exponential correlation function is used, better results are obtained for the like-polarized term. It can easily be shown, however, that a pure exponential correlation function leads to an imaginary surface slope distribution so that results using this correlation function are very misleading. The depolarized components from the tangent-plane method depend upon the sum of the two Fresnel reflection coefficients which does not compare well with experimental results in general.

Many natural surfaces can be assumed to be made of large undulations with small perturbations superimposed. Therefore, a correct theory on the backscattering of electromagnetic waves from rough surfaces should consider both types of surface roughness.

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[2] Beckmann and Spizzichino, *The Scattering of Electromagnetic Waves from Rough Surfaces*, Pergamon Press 1963, p. 29.

A recent paper by Fung and Hsias-Lien Chan[3] has done this for an incident wave with horizontal polarization by using the small perturbation method up to the first-order terms as an approximation of the fields on the surface. When the zero-order terms of the small perturbation solution are used as estimates of the surface fields and placed in the Stratton-Chu integral, the tangent-plane solution results. The paper by Fung and Chan did not consider the effects of along-track surface slopes or the effects of higher order slope terms in the power calculation. Our purpose here was to apply Fung's ideas such that the effects of along-track slopes and higher order slope terms are considered in the horizontal polarization solution and to perform the entire solution for an incident wave with vertical polarization. For the vertical polarization solution, it was necessary, first, to solve the problem of a vertically polarized wave being scattered from a slightly rough, dielectric surface. This derivation is performed in Appendix A. In the case of horizontal polarization, use was made of Rice's result[4] as derived by Fung.[5]

In practically all problems concerning the scattering of electromagnetic waves from rough surfaces, certain assumptions must be made in order to make the solution analytically manageable. The present solution is no exception. Some of the more basic assumptions which were made are:

(1) Shadowing effects are neglected.

(2) Only the far field is calculated.

(3) Multiple scattering is neglected.

(4) The density of the scattering elements is not considered.

(5) The treatment is restricted to surfaces that are generated by stationary random processes and which yield the elevation values distributed with a two-dimensional gaussian density distribution function.

In addition to the above assumptions, certain mathematical assumptions are made which allow an analytical expression to be obtained. The main assumption of the tangent-plane method in computing the fields at the surface was not used.

The derivations will be preceded by a discussion of the geometry of the problem to be solved. Following this, the Stratton-Chu integral will be introduced and modified for future calculations. Next, the entire scattering problem is derived for the case of a horizontally polarized incident plane wave. The final result is the scattering coefficient σ°

[3] A. K. Fung and Hsias-Lien Chan, "Backscattering of Waves by Composite Rough Surfaces," *IEEE Transactions on Antenna and Propagation*, September 1969.

[4] S. O. Rice, "Reflection of Electromagnetic Waves from Slightly Rough Surfaces," *Communications on Pure and Applied Mathematics*, Vol. 4, 1951.

[5] A. K. Fung, "Mechanisms of Polarized and Depolarized Scattering from a Rough Dielectric Surface," *Journal of the Franklin Institute*, Vol. 285, No. 2, February 1968.

for both the like and depolarized terms. The same problem is then solved for the case of a vertically polarized incident plane wave. The final results appear as graphs of σ° versus incident angle for various types of soils, moisture contents, surface-roughness parameters, and frequencies. A discussion of the final results and their possible meaning concludes the report. Throughout the report, the rationalized MKS system of units is used. The notation used in this report will be the same as that employed by Fung and Hsias-Lien Chan[6] in their composite surface theory so that the two results can be compared.

II. ANALYSIS

3. **Geometry of Scattering Problem.** A plane electromagnetic wave with a harmonic time dependence of $\exp(j\omega t)$ is incident onto a rough surface described by a function $\rho(x,y)$. This function $\rho(x, y)$ represents the total elevation from a mean plane to the surface at an arbitrary point (x, y). The total surface shall be composed of the sum of large undulations and a slightly rough surface. The large undulations shall be represented by the general function $Z(x, y)$ and shall be required to conform to the tangent-plane method. The slightly rough surface shall be designated by $s(x, y)$ and will be required to conform to the requirements of the small perturbation method. The relation among these surface functions is then $\rho(x,y) = Z(x, y) + s(x, y)$. It will be assumed that $Z(x, y)$ and $s(x,y)$ are generated by independent stationary random processes with zero means. The average values of $Z(x, y)$ and $s(x, y)$ are then the xy-plane. The geometry of the scattering problem is given in Fig. 2. A right-handed rectangular Cartesian coordinate system is set up near the surface such that the xy-plane is the mean surface height or elevation. The unit vector $\vec{n}_1$ is a vector in the direction of the incident wave. The angle θ is the incidence angle. The vector $\vec{r}$ is a range vector from the origin of the coordinate system to a surface point. The unit vector $\vec{n}$ is a normal to the total surface of $\rho(x,y)$. The field point P is where the scattered field is to be computed. The unit vector $\vec{R}_1$ points from the origin of the coordinate system to the field point P. The unit vector $\vec{n}_2$ is in the direction of the field point P from a surface point. When the point P is placed in the far field, it can be seen that $\vec{n}_2 = \vec{R}_1$. For the case of backscattering, $\vec{n}_2 = -\vec{n}_1$. The medium above the surface is assumed to be free space. The medium below the surface is a dielectric with zero conductivity. This is a good approximation for many soils especially at high radar frequencies. Another right-handed rectangular Cartesian coordinate system $(\bar{x}, \bar{y}, \bar{z})$ is set up with the origin sitting on the large undulations $Z(x, y)$ such that the $\bar{x}\bar{y}$-plane is tangent to $Z(x,y)$ at an arbitrary

[6] A. K. Fung and Hsias-Lien Chan.

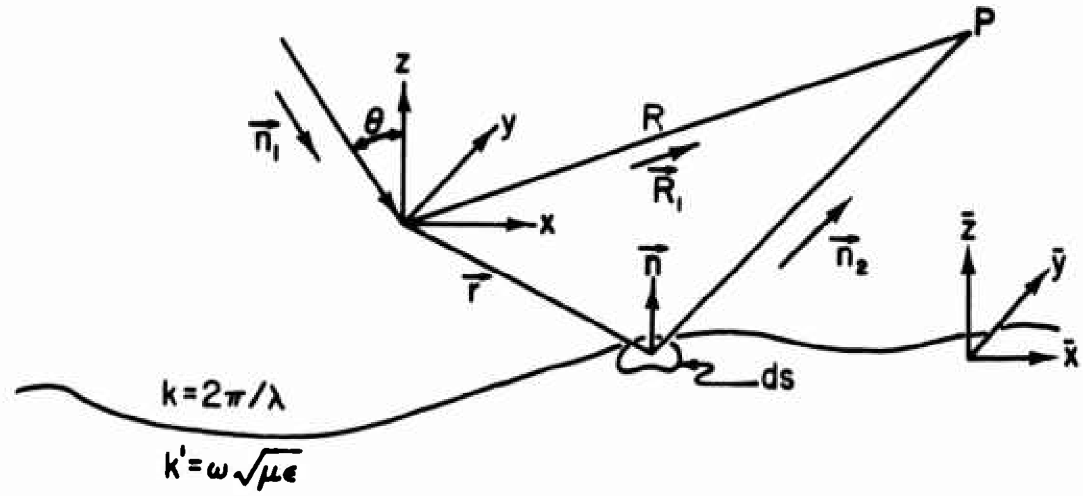

Fig. 2. Geometry of the scattering problem.

point. The $\bar{z}$-axis is then normal to the large undulations. The propagation constant in the medium $z < \rho(x, y)$ will be designated k' and is equal to $\omega\sqrt{\mu\epsilon}$. The parameters inside the radical sign represent the permeability and permittivity, respectively, of the material in the medium below the surface.

The scattered field in the direction defined by the unit vector $\vec{n}_2$ is then obtained from the Stratton-Chu integral as stated by Silver:[7]

$$\vec{E}_s = K\vec{n}_2 \times \int [\vec{n} \times \vec{E} - \eta\vec{n}_2 \times (\vec{n} \times \vec{H})] \exp(jk\vec{n}_2 \cdot \vec{r})\, ds . \tag{1}$$

$\vec{E}$ and $\vec{H}$ are the total electric and magnetic fields on the surface and η is the intrinsic impedance of the medium above the surface. The vector $\vec{n}$ is normal to the total surface.

$$K = \frac{-jke^{-jkR}}{4\pi R}$$

R is the distance from the origin to the point P. The parameter k is the wavenumber in free space and is equal to $2\pi/\lambda$ where λ is the wavelength. All quantities are known in equation (1) except the two surface-field vectors. The surface fields can be written in terms of the local coordinate system $(\bar{x}, \bar{y}, \bar{z})$ as follows:

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[7] Samuel Silver, "Microwave Antenna Theory and Design," *MIT Rad. Lab. Series 12*, McGraw-Hill, 1947, p. 161.

$$\vec{E} = \vec{\bar{x}}E_{\bar{x}} + \vec{\bar{y}}E_{\bar{y}} + \vec{\bar{z}}E_{\bar{z}}$$

$$\vec{H} = \vec{\bar{x}}H_{\bar{x}} + \vec{\bar{y}}H_{\bar{y}} + \vec{\bar{z}}H_{\bar{z}}$$

where $\vec{\bar{x}}, \vec{\bar{y}}, \vec{\bar{z}}$ are unit vectors in the local coordinate system $(\bar{x}, \bar{y}, \bar{z})$. The surface field components $E_{\bar{x}}, E_{\bar{y}}, E_{\bar{z}}$ are in terms of the local coordinates. The first problem which must be solved is to obtain $\vec{\bar{x}}, \vec{\bar{y}}$, and $\vec{\bar{z}}$ in terms of $\vec{i}, \vec{j}$, and $\vec{k}$ which are unit vectors in the (x, y, z) system. The surface functions will no longer be written with their (x, y) arguments:

$$\vec{\bar{z}} = (-\vec{i}Z_x - \vec{j}Z_y + \vec{k})(1 + Z_x^2 + Z_y^2)^{-1/2} \approx (-\vec{i}Z_x - \vec{j}Z_y + \vec{k})$$

where $Z_x = \frac{\partial Z}{\partial x}$ and $Z_y = \frac{\partial Z}{\partial y}$. These terms are representative of surface slopes in x and y. For small slopes, the term $Z_x^2 + Z_y^2$ is to be considered small with respect to one. Throughout this report, terms involving Z_x^2 and Z_y^2 will be considered negligible with respect to terms involving Z_x and Z_y. The direction that the $\bar{y}$-axis takes is defined by the unit vectors $\vec{n}_1$ and $\vec{\bar{z}}$:

$$\vec{\bar{y}} = (\vec{\bar{z}} \times \vec{n}_1)/|\vec{\bar{z}} \times \vec{n}_1|$$

$$\vec{\bar{x}} = \vec{\bar{y}} \times \vec{\bar{z}}$$

$$\vec{n}_1 = \vec{i}\sin\theta - \vec{k}\cos\theta \ .$$

By defining the local coordinate system $(\bar{x}, \bar{y}, \bar{z})$ in the above manner, a local plane of incidence has been established which is defined by the vectors $\vec{n}_1$ and $\vec{\bar{z}}$. It can be seen that this local plane of incidence does not, in general, coincide with the original plane of incidence defined by the unit vectors $\vec{n}_1$ and $\vec{k}$. The two planes will only coincide when Z_y is equal to zero. The local angle of incidence (θ') is defined as the angle between $\vec{\bar{z}}$ and $-\vec{n}_1$, and can be computed as follows:

$$\cos\theta' = -\vec{n}_1 \cdot \vec{\bar{z}} \approx \cos\theta + Z_x \sin\theta \ .$$

The unit vectors $\vec{\bar{y}}$ and $\vec{\bar{x}}$ can be computed:

$$\vec{\bar{z}} \times \vec{n}_1 = [\vec{i}Z_y\cos\theta + \vec{j}(\sin\theta - Z_x\cos\theta) + \vec{k}Z_y\sin\theta]$$

$$|\vec{\bar{z}} \times \vec{n}_1| \approx \sin\theta - Z_x\cos\theta$$

$$\vec{\bar{y}} \approx \vec{i}Z_y D_o\cos\theta + \vec{j} + \vec{k}Z_y D_o\sin\theta$$

$$\vec{\bar{x}} \approx \vec{i} - \vec{j}Z_y D_o\cos\theta + \vec{k}Z_x$$

where $D_o \approx (\sin\theta - Z_x \cos\theta)^{-1}$ and a Z_y^2 term has been dropped in D_o. The term $\vec{n} \times \vec{E}$ can be calculated in terms of the surface field components $E_{\bar{x}}$, $E_{\bar{y}}$, $E_{\bar{z}}$ and the unit vectors $\vec{i}, \vec{j}$, and $\vec{k}$:

$$\vec{n} \times \vec{E} = \vec{i}\,[E_{\bar{x}} Z_y D_o \cos\theta - E_{\bar{y}} - E_{\bar{z}} s_y] + \vec{j}\,[E_{\bar{x}} + E_{\bar{y}} Z_y D_o \cos\theta + E_{\bar{z}} s_x]$$

$$+ \vec{k}\,[\rho_y E_{\bar{x}} - \rho_x E_{\bar{y}}] .$$

The subscripts on the variables s and ρ refer to partial derivatives with respect to the subscripted variable. The term $\vec{n} \times \vec{H}$ can be written directly:

$$\vec{n} \times \vec{H} = \vec{i}\,[H_{\bar{x}} Z_y D_o \cos\theta - H_{\bar{y}} - H_{\bar{z}} s_y] + \vec{j}\,[H_{\bar{x}} + H_{\bar{y}} Z_y D_o \cos\theta + H_{\bar{z}} s_x]$$

$$+ \vec{k}\,[\rho_y H_{\bar{x}} - \rho_x H_{\bar{y}}].$$

In the case of backscattering, the term $\vec{n}_2 \times (\vec{n} \times \vec{E})$ is calculated as $-\vec{n}_1 \times (\vec{n} \times \vec{E})$:

$$-\vec{n}_1 \times (\vec{n} \times \vec{E}) = -(\vec{i}\cos\theta + \vec{k}\sin\theta)(E_{\bar{x}} + E_{\bar{y}} Z_y D_o \cos\theta + E_{\bar{z}} s_x)$$

$$+\vec{j}\,[E_{\bar{x}} (Z_y D_o \cos^2\theta + \rho_y \sin\theta) - E_{\bar{y}} (\cos\theta + \rho_x \sin\theta)$$

$$- E_{\bar{z}} s_y \cos\theta] .$$

The final term to be computed is $\vec{n}_2 \times (\vec{n}_2 \times (\vec{n} \times \vec{H}))$: $\vec{n}_2 \times (\vec{n}_2 \times (\vec{n} \times \vec{H})) = -\vec{j}\,[H_{\bar{x}} + H_{\bar{y}} Z_y D_o \cos\theta + H_{\bar{z}} s_x] - (\vec{i}\cos\theta + \vec{k}\sin\theta)[H_{\bar{x}} (Z_y D_o \cos^2\theta + \rho_y \sin\theta) - H_{\bar{y}} (\cos\theta + \rho_x \sin\theta) - H_{\bar{z}} s_y \cos\theta]$. The above terms can be placed in the Stratton-Chu integral to obtain an expression for the backscattered field in terms of the surface field components in local coordinates:

$$\begin{aligned}\vec{E}_s = K \iint \Big\{ & \vec{j}\,[E_{\bar{x}} Z_y D_o \cos^2\theta - E_{\bar{y}} \cos\theta - E_{\bar{z}} s_y \cos\theta + \eta H_{\bar{x}} \\ & + \eta H_{\bar{y}} Z_y D_o \cos\theta + \eta H_{\bar{z}} s_x + \sin\theta\,(\rho_y E_{\bar{x}} - \rho_x E_{\bar{y}})] \\ & - (\vec{i}\cos\theta + \vec{k}\sin\theta)[E_{\bar{x}} + E_{\bar{y}} Z_y D_o \cos\theta + E_{\bar{z}} s_x \\ & - \eta H_{\bar{x}} (Z_y D_o \cos^2\theta + \rho_y \sin\theta) + \eta H_{\bar{y}} (\cos\theta + \rho_x \sin\theta) \\ & + \eta H_{\bar{z}} s_y \cos\theta] \Big\} \exp(-jk\vec{n}_1 \cdot \vec{r})\, dy\, dx . \end{aligned} \tag{2}$$

The limits of integration are over the illuminated area. The only unknowns in equation (2) are the six surface field terms expressed in local coordinates. In order to proceed further, a particular incident polarization must be chosen and then the local fields calculated.

4. Horizontal Polarization.

a. Like-Polarized Term. A horizontally polarized plane wave with unit amplitude is incident onto the statistically rough dielectric (zero conductivity) surface $\rho(x, y)$. It shall be required to obtain analytical expressions for σ^o_{HH}, the scattering coefficient for the like-polarized return, and for σ^o_{HV}, the scattering coefficient for the depolarized return. In order to do this, an expression must first be obtained for E_{HH}, the like-polarized backscattered field. From this, the mean power density $< E_{HH} E^*_{HH} >$ must be calculated. Similarly, equations must be derived for E_{HV} and $< E_{HV} E^*_{HV}>$ in order to calculate σ^o_{HV}. The brackets are here used to denote the fact that an average of the quantity inside is being taken. The asterisk is used to indicate the complex conjugate. σ^o_{HH} can then be calculated from the equation

$$\sigma^o_{HH} = \lim_{R \to \infty} \frac{4\pi R^2 < E_{HH} E^*_{HH} >}{E_i E_i^*}$$

Where E_i is the value of the incident field without regard to polarization. The same method can be used to determine σ^o_{HV}. The incident wave for horizontal polarization can be written as

$$\vec{E}_i = \vec{j} \exp\left\{-jk(x\sin\theta - z\cos\theta)\right\} .$$

The time harmonic of exp ($j\omega t$) has been suppressed and will not be carried along any further.

In order to determine the surface fields in local coordinates, the incident wave must first be written in terms of the local coordinates. The locally incident field $\vec{E}_i^l$ is then

$$\vec{E}_i^l = [\vec{x}(\vec{x}\cdot\vec{j}) + \vec{y}(\vec{y}\cdot\vec{j}) + \vec{z}(\vec{z}\cdot\vec{j})] \exp\left\{-jk(x\sin\theta - Z\cos\theta)\right\} .$$

$$\exp\left\{-jk[(\vec{n}_i \cdot \vec{x})\overline{x} + (\vec{n}_i \cdot \vec{y})\overline{y} + (\vec{n}_i \cdot \vec{z})\overline{z}\right\} .$$

The quantities x and Z represent the coordinates of the origin of the local coordinate system. The original polarization of the wave has been broken up into three components in the local coordinate system. The above dot products can be easily calculated from results already derived:

$$\vec{x} \cdot \vec{j} = -Z_y D_o \cos\theta$$

$$\vec{y} \cdot \vec{j} = 1$$

$$\vec{z} \cdot \vec{j} = -Z_y .$$

Since $\vec{y} \cdot \vec{j} = 1$, and this is the magnitude of the original transmitted field vector, the effects of the $\vec{x} \cdot \vec{j}$ and the $\vec{z} \cdot \vec{j}$ components are neglected. This should be a very good approximation for small Z_y which must be the case in order for the tangent-plane method to be applicable to the smoothly undulating surface. The quantities $\vec{n}_1 \cdot \vec{x}$, $\vec{n}_1 \cdot \vec{y}$, and $\vec{n}_1 \cdot \vec{z}$ are easily computed from the previous work:

$$\vec{n}_1 \cdot \vec{x} = \sin\theta - Z_x \cos\theta$$

$$\vec{n}_1 \cdot \vec{y} = [Z_y \cos\theta \sin\theta - Z_y \cos\theta \sin\theta]/(\sin\theta - Z_x \cos\theta) = 0$$

$$\vec{n}_1 \cdot \vec{z} = -\cos\theta - Z_x \sin\theta \quad .$$

The local angle of incidence (θ') can be related to angle θ by the following expressions:

$$\cos\theta' \approx \cos\theta + Z_x \sin\theta$$

$$\sin\theta' \approx \sin\theta - Z_x \cos\theta \quad .$$

Using the expressions above for the local angle of incidence, the local incident field can be written as

$$\vec{E}_i^1 \approx \vec{y} \exp\{-jk(x\sin\theta - Z\cos\theta)\} \exp\{-jk(\bar{x}\sin\theta' - \bar{z}\cos\theta')\} .$$

The problem locally is now one of a horizontally polarized plane wave incident onto a slightly rough dielectric surface. This problem has been solved previously by Rice[8] and Fung.[9] The fields up to first order in perturbation, as expressed by Fung in the Fourier transform notation, will be the form used here:

$$E_{\bar{x}} = \left\{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_y k_x Q' \overline{EXP}\, dk_x\, dk_y \right\} EXP$$

$$E_{\bar{y}} = \left\{ \exp(-jk\bar{x}\sin\theta')\left[\exp(jk\bar{z}\cos\theta') + R_{\perp}\exp(-jk\bar{z}\cos\theta')\right] \right.$$

$$\left. - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (k_x^2 + k_z k_z') Q' \overline{EXP}\, dk_x\, dk_y \right\} EXP$$

[8] S. O. Rice.

[9] A. K. Fung, "Mechanisms . . . ."

$$E_{\bar{z}} = \left\{ -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_y k_z' Q' \,\overline{EXP}\, dk_x \, dk_y \right\} EXP \qquad (3)$$

$$Q' = \frac{jT(k'^2 - k^2)\, S(k_x + k\sin\theta', k_y)}{2\pi (k_z + k_z')(k_y^2 + k_x^2 + k_z k_z')}$$

$$T = 1 + R_\perp \; .$$

$R_\perp \equiv$ Fresnel reflection coefficient for a local horizontally polarized plane wave.

$$EXP = \exp\left[-jk(x\sin\theta - Z\cos\theta)\right]$$

$$\overline{EXP} = \exp\left[jk_x\bar{x} + jk_y\bar{y} - jk_z\bar{z}\right]$$

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2} \quad \text{when } k^2 > k_x^2 + k_y^2$$

$$k_z = -j\sqrt{k_x^2 + k_y^2 - k^2} \quad \text{when } k_x^2 + k_y^2 > k^2$$

$$k_z' = \sqrt{k'^2 - k_x^2 - k_y^2} \quad \text{when } k'^2 > k_x^2 + k_y^2$$

$$k_z' = -j\sqrt{k_x^2 + k_y^2 - k'^2} \quad \text{when } k_x^2 + k_y^2 > k'^2$$

$S(k_x, k_y)$ is the Fourier transform of the slightly rough surface function $s(x, y)$ with k_x, k_y as the Fourier variables:

$$S(k_x, k_y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(x,y)\exp(-jk_x x - jk_y y)\, dy\, dx \; .$$

It should be noticed that the k_x argument in $S(k_x, k_y)$ has been replaced by $k_x + k\sin\theta'$ in Q'. The angle θ' represents the local angle of incidence and is the angle between the unit vectors $\hat{z}$ and $-\hat{n}_1$. The Fresnel reflection coefficient $R_\perp$ is in terms of the local coordinate system and can be written as follows:

$$R_\perp = \frac{k\cos\theta' - \sqrt{k'^2 - k^2\sin^2\theta'}}{k\cos\theta' + \sqrt{k'^2 - k^2\sin^2\theta'}}$$

In place of $\cos\theta'$ and $\sin\theta'$, the following expressions are used:

$$\cos\theta' \approx \cos\theta + Z_x \sin\theta$$

$$\sin\theta' \approx \sin\theta - Z_x \cos\theta \ .$$

The present form of $R_\perp$ would make future calculations very difficult due to the Z_x in the radical and in the denominator. It would be advantageous to approximate $R_\perp$ with the first two terms of a Taylor series expansion about $Z_x = 0$ since small slopes are assumed. When this is done, $R_\perp$ can be written as

$$R_\perp \approx R_o + gZ_x$$

where $$R_o = \frac{k\cos\theta - \sqrt{k'^2 - k^2\sin^2\theta}}{k\cos\theta + \sqrt{k'^2 - k^2\sin^2\theta}}$$

$$g = -2k\,R_o \sin\theta / k'\cos\phi$$

$$\cos\phi = \sqrt{k'^2 - k^2\sin^2\theta}/k'$$

The angle ϕ would be the angle of refraction if the surface were flat, e.g., $\rho(x, y) = 0$. The angle ϕ is then related to the angle of incidence θ by Snell's Law. The magnetic fields associated with the electric fields given by equation (3) can be computed from Maxwell's equation written in the local coordinate system:

$$\eta H_{\bar{x}} = \Big\{ \exp(-jk\bar{x}\sin\theta')[\exp(jk\bar{z}\cos\theta') - R_\perp \exp(-jk\bar{z}\cos\theta')] \cdot \cos\theta'$$

$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ \frac{k_z(k_x^2 + k_z k_z')}{k} + \frac{k_y^2 k_z'}{k} \right\} Q' \,\overline{EXP}\, dk_x\, dk_y \Big\}\ EXP$$

$$\eta H_{\bar{y}} = \left\{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{k_y k_x}{k} (k_z - k_z')\, Q' \,\overline{EXP}\, dk_x\, dk_y \right\} EXP$$

$$\eta H_{\bar{z}} = \Big\{ \sin\theta' \exp(-jk\bar{x}\sin\theta')[\exp(jk\bar{z}\cos\theta') + R_\perp \exp(-jk\bar{z}\cos\theta')]$$

$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ \frac{k_x(k_x^2 + k_y^2 + k_z k_z')}{k} \right\} Q' \,\overline{EXP}\, dk_x\, dk_y \Big\}\ EXP \ .$$

At the surface, where the fields must be evaluated, the $\bar{x}$ and $\bar{y}$ coordinates are zero and $\bar{z} \approx s(x,y)$. The term Q' can also be written as

$$Q' = (1 + R_o + gZ_x)\,QS(k_x + k\sin\theta', k_y)$$

with $$Q = \frac{j(k'^2 - k^2)}{2\pi(k_z + k'_z)(k_y^2 + k_x^2 + k_z k'_z)} \ .$$

The fields evaluated on the surface then become

$$E_{\bar{x}} = \left\{ (1 + R_o) \int_{-\infty}^{\infty}\!\!\int_{-\infty}^{\infty} k_y k_x QS \exp(-jk_z s)\,dk_x\,dk_y + gZ_x \int_{-\infty}^{\infty}\!\!\int_{-\infty}^{\infty} k_y k_x QS \exp(-jk_z s)\,dk_x\,dk_y \right\} EXP$$

$$E_{\bar{y}} = \Big\{ \exp(jks\cos\theta) + R_o \exp(-jks\cos\theta) + gZ_x \exp(-jks\cos\theta)$$

$$- (1 + R_o) \int_{-\infty}^{\infty}\!\!\int_{-\infty}^{\infty} (k_x^2 + k_z k'_z)\,QS \exp(-jk_z s)\,dk_x\,dk_y$$

$$- gZ_x \int_{-\infty}^{\infty}\!\!\int_{-\infty}^{\infty} (k_x^2 + k_z k'_z)\,QS \exp(-jk_z s)\,dk_x\,dk_y \Big\} EXP$$

$$E_{\bar{z}} = -\left\{ (1 + R_o) \int_{-\infty}^{\infty}\!\!\int_{-\infty}^{\infty} k_y k'_z QS \exp(-jk_z s)\,dk_x\,dk_y + gZ_x \int_{-\infty}^{\infty}\!\!\int_{-\infty}^{\infty} k_y k'_z QS \exp(-jk_z s)\,dk_x\,dk_y \right\} EXP$$

$$\eta H_{\bar{x}} = \Big\{ \cos\theta \exp(jks\cos\theta) + Z_x \sin\theta \exp(jks\cos\theta) - R_o \cos\theta \exp(-jks\cos\theta)$$

$$- gZ_x \cos\theta \exp(-jks\cos\theta) - R_o Z_x \sin\theta \exp(-jks\cos\theta)$$

$$+ (1 + R_o) \int_{-\infty}^{\infty}\!\!\int_{-\infty}^{\infty} \left[\frac{k_z(k_x^2 + k_z k'_z) + k_y^2 k'_z}{k} \right] QS \exp(-jk_z s)\,dk_x\,dk_y$$

$$+ gZ_x \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\frac{k_z(k_x^2 + k_z k_z') + k_y^2 k_z'}{k}\right] QS\exp(-jk_z s)dk_x\, dk_y \Big\} \text{EXP}$$

$$\eta H_y = \Big\{ (1 + R_o) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{k_y k_x}{k} (k_z - k_z') QS\exp(-jk_z s)\, dk_x\, dk_y$$

$$+ gZ_x \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{k_y k_x}{k} (k_z - k_z')\, QS\exp(-jk_z s) dk_x\, dk_y \Big\} \text{EXP}$$

$$\eta H_{\bar z} = \Big\{ \sin\theta \exp(jks\cos\theta) - Z_x \cos\theta \exp(jks\cos\theta) + R_o \sin\theta \exp(-jks\cos\theta)$$

$$- R_o Z_x \cos\theta \exp(-jks\cos\theta) + gZ_x \sin\theta \exp(-jks\cos\theta)$$

$$+ (1 + R_o) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{\frac{k_x(k_x^2 + k_y^2 + k_z k_z')}{k}\right\} QS\exp(-jk_z s) dk_x\, dk_y$$

$$+ gZ_x \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{k_x(k_x^2 + k_y^2 + k_z k_z')}{k} QS\exp(-jk_z s) dk_x\, dk_y \Big\} \text{EXP} \quad . \qquad (4)$$

The parameter $S(k_x + k\sin\theta, k_y)$ has been written without its arguments. The $\vec{j}$ component of equation (2) for $\vec{E}_s$ together with the above equations for the surface fields can be used to calculate E_{HH}, the like-polarized backscattered field for a horizontally polarized incident wave. E_{HH} in terms of the surface field components is, from equation (2):

$$E_{HH} = K \iint [E_{\bar x} Z_{\bar y} D_o \cos^2\theta - E_{\bar y} \cos\theta - E_{\bar z}\, s_y \cos\theta + \eta H_{\bar x}$$

$$+ \eta H_{\bar y} Z_y D_o \cos\theta + \eta H_{\bar z}\, s_x + \sin\theta\, \rho_y E_{\bar x}$$

$$- \sin\theta\, \rho_x E_{\bar y}\,] \exp(-jk\vec{n}_i \cdot \vec{r}) dy\, dx \,.$$

When the surface field equations (4) are placed in the above expression for E_{HH}, the following equation results:

$$E_{HH} = -2R_o K \iint [\cos\theta + g_o Z_x \sin\theta]\ \exp(-jk\vec{n}_1 \cdot \vec{r})\ \mathrm{EXP}\exp(-jks\cos\theta)\,dy\,dx$$

$$+ I_{HH}$$

$$g_o = 1 - \frac{2k\cos\theta}{k'\cos\phi}$$

$$I_{HH} = K \iiiint \left\{ A + BZ_x + Cs_x + DZ_y + Es_y \right\} S\exp(-jk_z s)\,\mathrm{EXP}\exp(-jk\vec{n}_1 \cdot \vec{r}) \cdot$$

$$dy\,dx\,dk_x\,dk_y$$

where $A = (1+R_o)Q\cos\theta\,(k_x^2 + k_z k_z') + (1+R_o)\,Q\left[\frac{k_z(k_x^2 + k_z k_z')}{k} + \frac{k_y^2 k_z'}{k}\right]$

$$B = g\cos\theta\,(k_x^2 + k_z k_z')\,Q + g\left[\frac{k_z(k_x^2 + k_z k_z')}{k} + \frac{k_y^2 k_z'}{k}\right] Q$$

$$+ (1+R_o)\sin\theta\,(k_x^2 + k_z k_z')\,Q$$

$$C = (1+R_o)\left[\frac{k_x(k_x^2 + k_y^2 + k_z k_z')}{k}\right] Q + (1+R_o)\sin\theta\,(k_x^2 + k_z k_z')\,Q$$

$$D = (1+R_o)\ k_y k_x\,Q\csc\theta\cos^2\theta + \csc\theta\cos\theta\,(1+R_o)\left[\frac{k_y k_x}{k}(k_z - k_z')\right] Q$$

$$+ (1+R_o)\sin\theta\,k_y k_x\,Q$$

$$E = (1+R_o)\cos\theta\,k_y k_z'\,Q + (1+R_o)\sin\theta\,k_y k_x\,Q\ .$$

In the above calculations, the term D_o was approximated by the first term of its Taylor series expansion around $Z_x = 0$. The reason more terms of the series are not taken is that higher order slope terms would result in I_{HH} and these are assumed negligible. This results in the following approximation for D_o:

$$D_o \approx \csc\theta\ .$$

This approximation will be poor at very small angles of incidence but should be acceptable over a large range of incidence angles, particularly when the slopes are not large. The range vector $\vec{r}$ is

$$\vec{r} = \vec{i}x + \vec{j}y + \vec{k}\rho(x,y) = \vec{i}x + \vec{j}y + \vec{k}(Z + s).$$

Using the above expression for the range vector, the backscattered field E_{HH} becomes

$$E_{HH} = -K\iint [a_1 + b_1 Z_x] \exp[-2jkx\sin\theta + 2jkZ\cos\theta]\, dy\, dx + I_{HH}$$

$$a_1 = 2R_o \cos\theta \qquad b_1 = 2R_o g_o \sin\theta$$

$$I_{HH} = K\iiiint \left\{ A + BZ_x + Cs_x + DZ_y + Es_y \right\} S \exp(jas) \exp(-2jkx \sin\theta$$

$$+ 2jkZ\cos\theta)\, dy\, dx\, dk_x\, dk_y$$

$$a = k\cos\theta - k_z \ .$$

It is interesting to see that E_{HH} consists of the sum of two basic terms the first of which depends solely on the large surface undulations $Z(x,y)$ to which the tangent plane method applies and represents the quasi-specular component of the scattered field. The term I_{HH} arises because of the small surface perturbations $s(x,y)$; although it also contains terms dependent upon the large surface undulations. In calculating $E_{HH}\, E^*_{HH}$, it will be assumed that the two terms are independent so that cross-product terms would produce a negligible effect upon the final result:

$$E_{HH} E^*_{HH} = KK^* \iiiint \left\{ a_1^2 + a_1 b_1 Z_x + a_1 b_1 Z'_x + b_1^2 Z_x Z'_x \right\} .$$

$$\exp\left\{ -2jk\sin\theta(x - x') + 2jk\cos\theta(Z - Z') \right\} dy\, dx\, dy'\, dx'$$

$$+ I_{HH}\, I^*_{HH} \ .$$

In order to compute $\langle E_{HH} E^*_{HH} \rangle$, a two-dimensional distribution must be assumed for both $Z(x,y)$ and $s(x,y)$. The distributions to be used for both surfaces will be gaussian with zero means. The variance of the large undulations $Z(x,y)$ will be designated σ^2 . The variance of the slightly rough surface $s(x,y)$ is given the symbol σ_1^2 . With a two-dimensional gaussian distribution, not only must the mean and variance be known but the normalized autocorrelation function of the surfaces, called the autocorrelation coefficient, must be given a particular form. The symbol for the autocorrelation coefficient for the large undulations will be $C(r)$ and that for the slightly rough surface, $C_1(r)$. The form of $C(r)$ will be that suggested by Fung:[10]

$$C(r) = \exp\left\{\frac{-r^2}{L^2(G + r/L)}\right\}$$

$$r = \sqrt{(x - x')^2 + (y - y')^2}$$

$$x - x' \equiv u = r \sin\alpha$$

$$y - y' \equiv v = r \cos\alpha \quad .$$

It can be seen that $C(r)$ is a two-parameter correlation function, and when G is zero an exponential function results. The quantities L and G are physically significant in that they combine to give a measure of the distance between hills or valleys on the surface. With a large L or G, the distance between hills will be large. When G is small, $C(r)$ will be essentially exponential except near the origin, and this will result in a physically realizable surface with the variance of the surface slopes σ_s^2 being

$$\sigma_s^2 = -\sigma^2 C''(0) = \frac{2\sigma^2}{L^2 G} \quad .$$

The primes represent derivatives with respect to r . The form of $C_1(r)$ will be assumed gaussian and can be written as

$$C_1(r) = e^{-r^2/l^2} \quad .$$

The parameter l is the correlation distance. The following averages are computed in Appendix B and are needed to calculate $\langle E_{HH} E_{HH}^* \rangle$.

$$\langle \exp[2jk \cos\theta \, (Z - Z')] \rangle = \exp[-K_1(1 - C)]$$

$$\langle Z_x \exp[2jk \cos\theta \, (Z - Z')] \rangle = \langle Z'_x \exp[2jk \cos\theta \, (Z - Z')] \rangle$$

$$= -j2k\sigma^2 \cos\theta \, \frac{\partial C}{\partial u} \exp[-K_1(1 - C)]$$

$$\langle Z_x Z'_x \exp[2jk \cos\theta \, (Z - Z')] \rangle = -\sigma^2 \left[\frac{\partial^2 C}{\partial u^2} + K_1 \left(\frac{\partial C}{\partial u}\right)^2\right] \exp[-K_1(1 - C)]$$

[10] A. K. Fung, Correspondence to author, November 9, 1970.

$$K_1 = 4k^2\sigma^2\cos^2\theta \; .$$

The correlation coefficient has been written without its (u,v) arguments. Placing the averages in the integral results in the following:

$$\langle E_{HH}E_{HH}^*\rangle = KK^*\iint\Bigg\{a_1^2 - 4j\,a_1 b_1\, k\sigma^2\cos\theta\,\frac{\partial C}{\partial u}$$

$$- b_1^2\sigma^2\left[\frac{\partial^2 C}{\partial u^2} + K_1\left(\frac{\partial C}{\partial u}\right)^2\right]\Bigg\}\exp(-2jku\sin\theta)\exp[-K_1(1-C)]\,du\,dv$$

$$+\langle I_{HH}I_{HH}^*\rangle \; .$$

It will be easier to evaluate the above integral if the coordinates are changed from the rectangular (u,v) to the polar (r,α). Realizing that the Jacobian is equal to $-r$, the expression for $\langle E_{HH}E_{HH}^*\rangle$ now becomes:

$$\langle E_{HH}E_{HH}^*\rangle = KK^*\iint\Bigg\{a_1^2 - 4j\,a_1 b_1\, k\sigma^2\cos\theta\,\frac{\partial C}{\partial r}\sin\alpha$$

$$- b_1^2\sigma^2\left[\frac{\partial^2 C}{\partial r^2} + K_1\sin^2\alpha\left(\frac{\partial C}{\partial r}\right)^2\right]\Bigg\}r\exp(-2jkr\sin\alpha\sin\theta)\exp[-K_1(1-C)]\,dr\,d\alpha$$

$$+\langle I_{HH}I_{HH}^*\rangle \; .$$

When $K_1 >> 1$, a very rough surface results, and it can be seen that the main contribution to the integral in r comes only from the region about $r = 0$. The limits on r can then be made from zero to infinity. The limits on the integral in α will be from zero to 2π. The correlation function $C(r)$ can be approximated by taking the first two non-zero terms of its Taylor series expansion about $r = 0$. $C(r)$ then becomes

$$C(r) \approx 1 - \frac{r^2}{L^2 G}$$

or

$$C(u,v) \approx 1 - \frac{u^2+v^2}{L^2 G} \; .$$

Taking the indicated partial derivatives of $C(u,v)$ with respect to u yields the following expression for $\langle E_{HH}E_{HH}^*\rangle$:

$$<E_{HH}E_{HH}^*> = KK^*\int_0^{2\pi}\int_0^{\infty}\left\{a_1^2 + 8j\,a_1 b_1 k\cos\theta\,\frac{\sigma^2 r\sin\alpha}{L^2G} + \frac{2b_1^2\sigma^2}{L^2G}\right.$$

$$\left.\frac{-16b_1^2\sigma^4k^2\cos^2\theta\, r^2\sin^2\alpha}{L^4G^2}\right\}\exp(-2jkr\sin\theta\sin\alpha)\exp\left[\frac{-K_1r^2}{L^2G}\right] r\,dr\,d\alpha$$

$$+ <I_{HH}I_{HH}^*> .$$

In order to perform the integration in α, the following integrals are needed (see Appendix C):

$$\int_0^{2\pi}\exp(-2jkr\sin\theta\sin\alpha)\,d\alpha = 2\pi J_o(2kr\sin\theta)$$

$$\int_0^{2\pi}\sin\alpha\exp(-2jkr\sin\theta\sin\alpha)\,d\alpha = -j2\pi J_1(2kr\sin\theta)$$

$$\int_0^{2\pi}\sin^2\alpha\exp(-2jkr\sin\theta\sin\alpha)\,d\alpha = 2\pi\left\{\frac{J_1(2kr\sin\theta)}{2kr\sin\theta} - J_2(2kr\sin\theta)\right\} .$$

The J functions represent Bessel functions of the first kind:

$$<E_{HH}E_{HH}^*> = KK^*\int_0^{\infty}\left\{2\pi a_1^2 J_o(2kr\sin\theta) + \frac{16\pi a_1 b_1 k\cos\theta\,\sigma^2 rJ_1(2kr\sin\theta)}{L^2G}\right.$$

$$+ \frac{4\pi b_1^2\sigma^2}{L^2G}J_o(2kr\sin\theta) - \frac{32\pi b_1^2\sigma^4k^2\cos^2\theta\, r^2}{L^4G^2}\left[\frac{J_1(2kr\sin\theta)}{2kr\sin\theta}\right.$$

$$\left.\left. - J_2(2kr\sin\theta)\right]\right\}\exp\left[-\frac{K_1r^2}{L^2G}\right] rdr + <I_{HH}I_{HH}^*>$$

The integration in r can be carried out with the aid of the following integral:

$$\int_0^{\infty} x^{v+1}e^{-\alpha x^2}J_v(\beta x)\,dx = \frac{\beta^v}{(2\alpha)^{v+1}}\exp\left(\frac{-\beta^2}{4\alpha}\right)$$

The following expression is then obtained for $<E_{HH}E_{HH}^*>$:

$$<E_{HH}E^*_{HH}> = KK^*\left\{\frac{\pi a_1^2 L^2 G}{K_1} + \frac{8\pi\sigma^2 L^2 G a_1 b_1 k^2 \sin\theta\cos\theta}{K_1^2}\right.$$

$$\left. + \frac{4\pi L^2 G b_1^2 \sigma^2 k^2 \sin^2\theta}{K_1^2}\right\} \exp[-k^2 L^2 G \sin^2\theta / K_1] + <I_{HH}I^*_{HH}> \quad .$$

The quasi-specular term has been determined. The component $<I_{HH}I^*_{HH}>$ must now be calculated to complete the result. From previous work, an expression for $I_{HH}I^*_{HH}$ can be written

$$I_{HH}I^*_{HH} = KK^*\iiiint\iiiint \Big\{AA^* + AB^*Z'_x + AC^*s'_x + AD^*Z'_y$$

$$+ AE^*s'_y + BA^*Z_x + BB^*Z_xZ'_x + BC^*Z_xs'_x + BD^*Z_xZ'_y + BE^*Z_xs'_y$$

$$+ CA^*s_x + CB^*s_xZ'_x + CC^*s_xs'_x + CD^*s_xZ'_y + CE^*s_xs'_y$$

$$+ DA^*Z_y + DB^*Z_yZ'_x + DC^*Z_ys'_x + DD^*Z_yZ'_y + DE^*Z_ys'_y$$

$$+ EA^*s_y + EB^*s_yZ'_x + EC^*s_ys'_x + ED^*s_yZ'_y + EE^*s_ys'_y\Big\} \quad .$$

$$SS^* \exp(jas - ja^*s') \exp\left\{-2jk\sin\theta(x-x') + 2jk\cos\theta(Z-Z')\right\} \, .$$

$$dy\,dx\,dy'\,dx'\,dk_x\,dk_y\,dk'_x\,dk'_y \quad .$$

In order to calculate $<I_{HH}I^*_{HH}>$, it is necessary to use the following averages which are computed in Appendix B:

$$<Z_xZ'_y \exp[2jk\cos\theta(Z-Z')]> = <Z'_xZ_y \exp[2jk\cos\theta(Z-Z')]>$$

$$= -\sigma^2\left[\frac{\partial^2 C}{\partial u\partial v} + K_1\left(\frac{\partial C}{\partial u}\right)\left(\frac{\partial C}{\partial v}\right)\right]\exp[-K_1(1-C)]$$

$$<SS^* \exp(jas - ja^*s')> \approx 2\pi\delta(k'_x - k_x)\,\delta(k'_y - k_y)\,\sigma_1^2\,W(k_x + k\sin\theta, k_y)$$

$$\cdot \exp\left\{-\frac{1}{2}\sigma_1^2\,[a^2 + a^{*2} - 2aa^*C_1]\right\}$$

$$<\exp[2jk\cos\theta(Z-Z')]> = \exp[-K_1(1-C)]$$

$$<Z_x \exp[2jk\cos\theta(Z - Z')]> = <Z'_x \exp[2jk\cos\theta(Z - Z')]>$$

$$= -j2k\sigma^2 \cos\theta \left(\frac{\partial C}{\partial u}\right) \exp[-K_1(1 - C)]$$

$$<Z_y \exp[2jk\cos\theta(Z - Z')]> = <Z'_y \exp[2jk\cos\theta(Z - Z)]>$$

$$= -j2k\sigma^2 \cos\theta \left(\frac{\partial C}{\partial v}\right) \exp[-K_1(1 - C)]$$

$$<s_x SS^* \exp(jas - ja^*s')> \approx -ja^*\sigma_1^4 \frac{\partial C_1}{\partial u} 2\pi\delta(k'_x - k_x)\delta(k'_y - k_y)$$

$$\cdot W(k_x + k\sin\theta, k_y)\exp\left\{-\frac{1}{2}\sigma_1^2 [a^2 + a^{*2} - 2aa^*C_1]\right\}$$

$$<s'_x SS^* \exp(jas - ja^*s')> \approx -ja\sigma_1^4 \frac{\partial C_1}{\partial u} 2\pi\delta(k'_x - k_x)\delta(k'_y - k_y)$$

$$\cdot W(k_x + k\sin\theta, k_y)\exp\left\{-\frac{1}{2}\sigma_1^2 (a^2 + a^{*2} - 2aa^*C_1)\right\}$$

$$<s_y SS^* \exp(jas - ja^*s')> \approx -ja^*\sigma_1^4 \frac{\partial C_1}{\partial v} 2\pi\delta(k'_x - k_x)\delta(k'_y - k_y)$$

$$\cdot W(k_x + k\sin\theta, k_y)\exp\left\{-\frac{1}{2}\sigma_1^2 [a^2 + a^{*2} - 2aa^*C_1]\right\}$$

$$<s_x s'_x SS^* \exp(jas - ja^*s')> \approx -2\pi\sigma_1^4 \delta(k'_x - k_x)\delta(k'_y - k_y) W(k_x + k\sin\theta, k_y)$$

$$\cdot \left\{\frac{\partial^2 C_1}{\partial u^2} + \sigma_1^2 aa^* \left(\frac{\partial C_1}{\partial u}\right)^2\right\} \exp\left\{-\frac{1}{2}\sigma_1^2 [a^2 + a^{*2} - 2aa^*C_1]\right\}$$

$$<Z_y Z'_y \exp[2jk\cos\theta(Z - Z')]> = -\sigma^2 \left\{\frac{\partial^2 C}{\partial v^2} + K_1 \left(\frac{\partial C}{\partial v}\right)^2\right\} \exp[-K_1(1 - C)]$$

$$<s'_y SS^* \exp(jas - ja^*s')> \approx -j2\pi\sigma_1^4 a \frac{\partial C_1}{\partial v} \delta(k'_x - k_x)\delta(k'_y - k_y)$$

$$\cdot W(k_x + k\sin\theta, k_y)\exp\left\{-\frac{1}{2}\sigma_1^2 [a^2 + a^{*2} - 2aa^*C_1]\right\}$$

$$<s_x s'_y SS^* \exp(jas - ja^*s')> \approx -2\pi\delta(k'_x - k_x)\delta(k'_y - k_y)\sigma_1^4 \cdot W(k_x + k\sin\theta, k_y)\left[\frac{\partial^2 C_1}{\partial u \partial v}\right.$$

$$+ \sigma_1^2 aa^* \left(\frac{\partial C_1}{\partial u}\right)\left(\frac{\partial C_1}{\partial v}\right)\Big] \exp\left\{-\frac{1}{2}\sigma_1^2 [a^2 + a^{*2} - 2aa^*C_1]\right\}$$

$$<s_y s_y' SS^* \exp(jas - ja^*s')> \approx -2\pi\delta(k_x' - k_x)\delta(k_y' - k_y)\sigma_1^4 \cdot W(k_x + k\sin\theta, k_y)\left[\frac{\partial^2 C_1}{\partial v^2}\right.$$

$$\left. + \sigma_1^2 aa^* \left(\frac{\partial C_1}{\partial v}\right)^2\right] \exp\left\{-\frac{1}{2}\sigma_1^2 [a^2 + a^{*2} - 2aa^*C_1]\right\} .$$

$W(k_x, k_y)$ is the roughness spectrum of $s(x,y)$ and is equal to the two-dimensional Fourier transform of $C_1(u,v)$. Using the form of $C_1(u,v)$ previously given, $W(k_x + k\sin\theta, k_y) = \frac{l^2}{2}\exp\left\{-([k_x + k\sin\theta]^2 + k_y^2)\frac{l^2}{4}\right\}$. When the above averages are used in computing $<I_{HH} I_{HH}^*>$ and the integration is carried out in k_x' and k_y', the delta functions will become one. Placing the expression given previously for $C_1(u,v)$ and $C(u,v)$ in the averages and changing the integral to polar coordinates results in

$$<I_{HH} I_{HH}^*> = KK^* \iiiint \Big\{ 2\pi\sigma_1^2 AA^* + (A^*B + B^*A) 8\pi jk\cos\theta\,\sigma^2 \frac{r\sin\alpha\,\sigma_1^2}{L^2G}$$

$$+ j(AC^*a + A^*Ca^*)\sigma_1^4 \frac{4\pi r\sin\alpha}{l^2} e^{-r^2/l^2} + (A^*D + AD^*) \cdot$$

$$\frac{j8k\cos\theta\,\sigma^2\, r\cos\alpha\,\pi\sigma_1^2}{L^2G}$$

$$+ (AE^*a + A^*Ea^*) j4\pi \frac{\sigma_1^4 r\cos\alpha e^{-r^2/l^2}}{l^2} + \frac{4\pi\sigma_1^2\sigma^2 BB^*}{L^2G} - \frac{8\pi\sigma_1^2\sigma^2 BB^*K_1 r^2\sin^2\alpha}{L^4G^2}$$

$$- 16\pi(BC^*a + B^*Ca^*) k\cos\theta\,\sigma^2 \frac{r^2\sin^2\alpha\,\sigma_1^4 e^{-r^2/l^2}}{L^2Gl^2}$$

$$- \frac{8\pi\sigma_1^2\sigma^2(B^*D + BD^*)K_1 r^2\sin\alpha\cos\alpha}{L^4G^2} - 16\pi\sigma_1^4\sigma^2 (BE^*a + B^*Ea^*) \cdot$$

$$\frac{k\cos\theta\, r^2\sin\alpha\cos\alpha e^{-r^2/l^2}}{L^2Gl^2}$$

$$+ \frac{4\pi\sigma_1^4 CC^*}{l^2}\left[1 - \frac{2r^2\sin^2\alpha}{l^2}\right] e^{-r^2/l^2} - 16\pi(CD^*a^* + C^*Da) \cdot$$

$$\frac{k\cos\theta\,\sigma^2 r^2 \sin\alpha\cos\alpha\,\sigma_1^4 e^{-r^2/l^2}}{l^2 L^2 G}$$

$$- \frac{(CE^* + C^*E)\,8\pi\sigma_1^4 r^2 \sin\alpha\cos\alpha\, e^{-r^2/l^2}}{l^4} + \frac{4\pi\sigma_1^2\sigma^2 DD^*}{L^2 G}$$

$$- \frac{8\pi\sigma_1^2\sigma^2 DD^* K_1 r^2\cos^2\alpha}{L^4 G^2} - (DE^*a + D^*Ea^*)\,\frac{16\pi\sigma_1^4 r^2\cos^2\alpha\, k\sigma^2\cos\theta\, e^{-r^2/l^2}}{l^2 L^2 G}$$

$$\left. + \frac{4\pi EE^*\sigma_1^4 e^{-r^2/l^2}}{l^2} - \frac{8\pi EE^*\sigma_1^4 r^2\cos^2\alpha\, e^{-r^2/l^2}}{l^4} \right\} W(k_x + k\sin\theta, k_y)\,\cdot$$

$$\cdot \exp\left\{-\frac{1}{2}\sigma_1^2(a^2 + a^{*2})\right\}\left\{1 + \sigma_1^2 aa^*C_1\right\}\exp\left[-K_1 r^2/L^2 G\right]\cdot$$

$$\exp(-2jkr\sin\theta\sin\alpha)\cdot r\,dk_x\,dk_y\,dr\,d\alpha \quad .$$

In the above expression, the term $\exp(\sigma_1^2 aa^*C_1)$ has been replaced with $1 + \sigma_1^2 aa^*C_1$. The final form of $<I_{HH} I_{HH}^*>$ must be left as a double integral in k_x and k_y . Integrations can be performed in α and r as with the quasi-specular term. In order to do this, a few more integrals must be specified:

$$\int_0^{2\pi} \cos\alpha \exp(-j2kr\sin\theta\sin\alpha)\,d\alpha = 0$$

$$\int_0^{2\pi} \cos\alpha\sin\alpha \exp(-j2kr\sin\theta\sin\alpha)\,d\alpha = 0$$

$$\int_0^{2\pi} \cos^2\alpha \exp(-j2kr\sin\theta\sin\alpha)\,d\alpha = \frac{2\pi J_1(2kr\sin\theta)}{2kr\sin\theta}$$

The integration in r can be carried out using the same Bessel function integral that was used earlier to evaluate the quasi-specular term. When the integration is carried out in α and r, the following equation results for $<I_{HH} I_{HH}^*>$:

$$<I_{HH} I_{HH}^*> = 4\pi^2\sigma_1^2 KK^* \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{\frac{AA^* L^2 G\exp[-k^2 L^2\sin^2\theta G/K_1]}{2K_1}\right.$$

$$+\frac{2k^2 L^2 G\sigma^2 \cos\theta \sin\theta\,(AB^* + A^*B) \exp\left[-k^2 L^2 G \sin^2\theta/K_1\right]}{K_1^2}$$

$$+\frac{\sigma_1^2 k \sin\theta\,(AC^*a + A^*Ca^*) \exp\left[-k^2 \sin^2\theta \Big/ \left[\frac{K_1}{L^2G} + \frac{1}{l^2}\right]\right]}{l^2\left[\frac{K_1}{L^2G} + \frac{1}{l^2}\right]^2}$$

$$+\frac{2L^2 G\sigma^2 BB^* k^2 \sin^2\theta \exp\left[-k^2 L^2 G \sin^2\theta/K_1\right]}{K_1^2} +$$

$$-\frac{2(BC^*a + B^*Ca^*)k\cos\theta\,\sigma^2\sigma_1^2 \exp\left[-k^2\sin^2\theta \Big/ \left[\frac{K_1}{L^2G} + \frac{1}{l^2}\right]\right]}{L^2Gl^2\left[\frac{K_1}{L^2G} + \frac{1}{l^2}\right]^2}$$

$$+\frac{4(BC^*a + B^*Ca^*)k^3\cos\theta\sin^2\theta\,\sigma^2\sigma_1^2 \exp\left\{-k^2\sin^2\theta \Big/ \left[\frac{K_1}{L^2G} + \frac{1}{l^2}\right]\right\}}{L^2Gl^2\left[\frac{K_1}{L^2G} + \frac{1}{l^2}\right]^3}$$

$$+\frac{\sigma_1^2 CC^* L^2 G l^2}{(K_1 l^2 + L^2 G)^2} \exp\left[\frac{-k^2\sin^2\theta}{\left(\frac{K_1}{L^2G} + \frac{1}{l^2}\right)}\right] \left[K_1 + \frac{2L^4G^2k^2\sin^2\theta}{(K_1 l^2 + L^2G)}\right]$$

$$-\frac{2(DE^*a + D^*Ea^*)\sigma_1^2\sigma^2 k\cos\theta \exp\left\{-k^2\sin^2\theta \Big/ \left[\frac{K_1}{L^2G} + \frac{1}{l^2}\right]\right\}}{l^2L^2G\left[\frac{K_1}{L^2G} + \frac{1}{l^2}\right]^2}$$

$$+\frac{\sigma_1^2 EE^* K_1 L^2 G l^2}{(K_1 l^2 + L^2 G)^2} \exp\left\{-k^2\sin^2\theta \Big/ \left[\frac{K_1}{L^2G} + \frac{1}{l^2}\right]\right\}$$

$$+\frac{\sigma_1^2 aa^* AA^* \exp\left\{-k^2\sin^2\theta \Big/ \left[\frac{K_1}{L^2G} + \frac{1}{l^2}\right]\right\}}{2\left[\frac{K_1}{L^2G} + \frac{1}{l^2}\right]}$$

$$+\frac{2(AB^* + A^*B)aa^*\sigma_1^2\sigma^2 k^2 \cos\theta \sin\theta \exp\left\{-k^2\sin^2\theta\Big/\left[\frac{K_1}{L^2G}+\frac{1}{l^2}\right]\right\}}{L^2G\left[\frac{K_1}{L^2G}+\frac{1}{l^2}\right]^2}$$

$$+\frac{aa^*BB^*\sigma_1^2\sigma^2L^2Gl^2}{(K_1 l^2 + L^2G)^2}\exp\left[\frac{-k^2\sin^2\theta}{\frac{K_1}{L^2G}+\frac{1}{l^2}}\right]\left[1+\frac{2k^2\sin^2\theta K_1 l^4}{(K_1 l^2 + L^2G)}\right]$$

$$+\frac{aa^*DD^*\sigma_1^2\sigma^2L^2Gl^2}{(K_1 l^2 + L^2G)^2}\exp\left[\frac{-k^2\sin^2\theta}{\frac{K_1}{L^2G}+\frac{1}{l^2}}\right]\Bigg\} W \exp\left\{-\frac{1}{2}\sigma_1^2(a^2+a^{*2})\right\}dk_x dk_y \;.$$

Terms containing σ_1^6 have been dropped. In order to obtain a numerical result for σ^o_{HH} as a function of θ, the double integral in $<I_{HH}I^*_{HH}>$ must be evaluated numerically.

b. Depolarized Term. Attention is now directed to the derivation of the depolarized scattering coefficient σ^o_{HV}. The basic expression needed to compute σ^o_{HV} is

$$\sigma^o_{HV} = \lim_{R\to\infty} \frac{4\pi R^2 <E_{HV}E^*_{HV}>}{E_iE_i^*} \;.$$

The objective now is to calculate the quantity $<E_{HV}E^*_{HV}>$. The $\vec{i}$ and $\vec{k}$ components from equation (2) form E_{HV}:

$$E_{HV} = -K\iint\Big\{E_{\bar{x}} + E_{\bar{y}}Z_y\csc\theta\cos\theta + s_xE_{\bar{z}} - \eta H_{\bar{x}}Z_y\csc\theta\cos^2\theta$$

$$-\eta H_{\bar{z}}Z_y\sin\theta - \eta H_{\bar{x}}s_y\sin\theta + \eta H_{\bar{y}}\cos\theta + \eta H_{\bar{y}}Z_x\sin\theta$$

$$+\eta H_{\bar{y}}s_x\sin\theta + \eta H_{\bar{z}}s_y\cos\theta\Big\}\exp(-jk\,\vec{n}_1\cdot\vec{r})\,dy\,dx \;.$$

Placing the surface fields from equations (4) into the above expression for E_{HV} results in the following equation:

$$E_{HV} = -K \iint \left\{ a_2 Z_y + b_2 s_y \right\} \exp\left[-2jkx \sin\theta + 2jkZ\cos\theta\right] dy\,dx + I_{HV}$$

where: $a_2 = 2R_o \csc\theta \cos\theta$; $b_2 = 2R_o \sin\theta \cos\theta$

$$I_{HV} = K \iiiint \left\{ A' + B'Z_x + C's_x + D'Z_y + E's_y \right\} S \exp(jas) \exp\left[-2jkx \sin\theta + 2jkZ\cos\theta\right] dk_x\, dk_y\, dy\, dx$$

$$A' = -Q(1+R_o)k_y k_x - Q(1+R_o)\cos\theta \frac{k_y k_x}{k}(k_z - k_z')$$

$$B' = -Qg k_y k_x - Qg\cos\theta \frac{k_y k_x}{k}(k_z - k_z') - Q(1+R_o)\sin\theta \frac{k_y k_x}{k}(k_z - k_z')$$

$$C' = Q(1+R_o)k_y k_z' - Q(1+R_o)\sin\theta \frac{k_y k_x}{k}(k_z - k_z')$$

$$D' = Q\csc\theta\cos\theta(1+R_o)(k_x^2 + k_z k_z') + Q(1+R_o)\csc\theta\cos^2\theta\left[\frac{k_z(k_x^2 + k_z k_z')}{k} + \frac{k_y^2 k_z'}{k}\right] + Q(1+R_o)\sin\theta\left[\frac{k_z(k_x^2 + k_z k_z')}{k} + \frac{k_y^2 k_z'}{k}\right]$$

$$E' = (1+R_o)Q\sin\theta\left[\frac{k_z(k_x^2 + k_z k_z')}{k} + \frac{k_y^2 k_z'}{k}\right] - Q(1+R_o)\left[\frac{k_x(k_x^2 + k_z k_z') + k_y^2 k_x}{k}\right]\cos\theta$$

It should be noted that I_{HV} has been written in the same form as I_{HH} and that only the definitions of the coefficients of the slope terms are different:

$$E_{HV}E_{HV}^* = KK^* \iiiint \left\{ a_2^2 Z_y Z_y' + a_2 b_2 Z_y s_y' + a_2 b_2 Z_y' s_y + b_2^2 s_y s_y' \right\} \cdot \exp\left\{-2jk\sin\theta(x - x') + 2jk\cos\theta(Z - Z')\right\} dy\,dx\,dy'\,dx' + I_{HV}I_{HV}^* .$$

In order to compute $\langle E_{HV}E_{HV}^* \rangle$, a few more averages are needed:

$$\langle Z_y Z_y' \exp(2jk\cos\theta(Z - Z'))\rangle = -\sigma^2\left[\frac{\partial^2 C}{\partial v^2} + K_1\left(\frac{\partial C}{\partial v}\right)^2\right]\exp\left\{-K_1(1 - C)\right\}$$

$$<s_y> = <s'_y> = 0$$

$$<s_y s'_y> = -\sigma_1^2 \frac{\partial^2 C_1}{\partial v^2} .$$

Using the above averages, the equation for $C_1(u,v)$ and the approximate equation for $C(u,v)$, the expression for $<E_{HV} E^*_{HV}>$ becomes

$$<E_{HV} E^*_{HV}> = KK^* \int_0^{2\pi}\int_0^{\infty} \left\{ \frac{2\sigma^2 a_2^2}{L^2 G} - \frac{4K_1 \sigma^2 r^2 a_2^2 \cos^2\alpha}{L^4 G^2} + \frac{2\sigma_1^2 b_2^2 e^{-r^2/l^2}}{l^2} \right.$$

$$\left. - \frac{4\sigma_1^2 r^2 b_2^2 \cos^2\alpha e^{-r^2/l^2}}{l^4} \right\} \exp(-2jkr \sin\theta \sin\alpha) \exp[-K_1(1 - C)]\, r\,dr\,d\alpha$$

$$+ <I_{HV} I^*_{HV}> .$$

The change from rectangular coordinates (u,v) to polar coordinates (r,α) has been made in the integral. The integrations in α and r can be carried out easily by use of the previously defined integrals and the approximate expression for $C(r)$. When these integrations are performed, the final expression for $<E_{HV} E^*_{HV}>$ is

$$<E_{HV} E^*_{HV}> = \frac{2\pi KK^* b_2^2 \sigma_1^2 K_1 L^2 G l^2}{(K_1 l^2 + L^2 G)^2} \exp\left(\frac{-k^2 \sin^2\theta}{\left(\frac{K_1}{L^2 G} + \frac{1}{l^2}\right)} \right) + <I_{HV} I^*_{HV}> .$$

The equation for $<I_{HV} I^*_{HV}>$ will be the same as that for $<I_{HH} I^*_{HH}>$ except that A, B, C, D, E will be replaced by A′, B′, C′, D′, E′, respectively. It is interesting to see that, although the depolarized scattering coefficient does depend on the large surface undulations, if $\sigma_1 = 0$, the scattering coefficient goes to zero also. A discussion of the results and their meaning will be given in a later section of the report.

5. Vertical Polarization.

a. Like-Polarized Term. Consider now a vertically polarized plane wave of unit amplitude with a time harmonic $\exp(j\omega t)$ incident upon the composite rough surface $\rho(x,y) = Z(x,y) + s(x,y)$. The surface is again assumed to be a dielectric with a gaussian distribution of surface heights. It will be required to derive expressions for σ^0_{VV}, the like-polarized backscatter coefficient, and σ^0_{VH}, the depolarized backscatter coefficient. In order to do this, expressions must be derived for E_{VV}, $<E_{VV} E^*_{VV}>$,

E_{VH}, and $<E_{VH} E^{*}_{VH}>$, the fields and mean power densities of the like-polarized and depolarized returns. Then, σ^{o}_{VV} and σ^{o}_{VH} can be determined by the following:

$$\sigma^{o}_{VV} = \lim_{R \to \infty} \frac{4\pi R^2 <E_{VV} E^{*}_{VV}>}{E_i E_i^*}$$

$$\sigma^{o}_{VH} = \lim_{R \to \infty} \frac{4\pi R^2 <E_{VH} E^{*}_{VH}>}{E_i E_i^*} .$$

For a vertically polarized wave, the incident electric field vector is written as

$$\vec{E}_i = -(\vec{i}\cos\theta + \vec{k}\sin\theta)\exp[-jk(x\sin\theta - z\cos\theta)] .$$

From Maxwell's equation, the incident magnetic field is

$$\vec{H}_i = \frac{\vec{j}\exp\{-jk(x\sin\theta - z\cos\theta)\}}{\eta}$$

where η is the intrinsic impedence of free space. For vertical polarization, it will be easier to work with the magnetic field vector rather than the electric field vector. The locally incident magnetic field vector $\vec{H}_i^{\,l}$ can be written in terms of the local coordinates $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ in the same manner as the incident electric vector was written in terms of local coordinates for horizontal polarization:

$$\vec{H}_i^{\,l} = \frac{1}{\eta}[\vec{x}(\vec{x}\cdot\vec{j}) + \vec{y}(\vec{y}\cdot\vec{j}) + \vec{z}(\vec{z}\cdot\vec{j})]\exp\left\{-jk(x\sin\theta - Z\cos\theta)\right\}$$

$$\cdot\exp\left\{-jk[\vec{n}_1\cdot\vec{x})\overline{x} + (\vec{n}_1\cdot\vec{y})\overline{y} + (\vec{n}_1\cdot\vec{z})\overline{z}]\right\}.$$

The above vector dot products have been determined previously and are here repeated for convenience:

$$\vec{x}\cdot\vec{j} = -Z_y D_o\cos\theta$$

$$\vec{y}\cdot\vec{j} = 1$$

$$\vec{z}\cdot\vec{j} = -Z_y .$$

Once again, since $\vec{j}\cdot\vec{y} = 1$ the effects of the $\vec{x}\cdot\vec{j}$ and $\vec{z}\cdot\vec{j}$ components will be neglected. Using the quantities $\vec{n}_1\cdot\vec{x}$, $\vec{n}_1\cdot\vec{y}$, and $\vec{n}_1\cdot\vec{z}$ as computed previously, the local incident magnetic field vector can be written as

$$H_i^i \approx \frac{1}{\eta}\,\hat{y}\exp\left\{-jk(x\sin\theta - Z\cos\theta)\right\}\exp\left\{-jk(\bar{x}\sin\theta' - \bar{z}\cos\theta')\right\}.$$

The problem locally near the surface is that of a vertically polarized wave incident onto a slightly rough dielectric surface with an angle of incidence equal to θ' . This problem is worked in Appendix A, and the results there are restated here in the local coordinate system. The fields are correct up to first-order terms:

$$H_{\bar{x}} = \left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D_{x1}(k_x,k_y)\,\overline{EXP}\,dk_y\,dk_x\right\} EXP$$

$$H_{\bar{y}} = \left\{\exp(-jk\bar{x}\sin\theta')\,[\exp(jk\bar{z}\cos\theta') + R_{\|}\exp(-jk\bar{z}\cos\theta')]\right.$$

$$\left. + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_{y1}(k_x,k_y)\,\overline{EXP}\,dk_x\,dk_y\right\} EXP$$

$$H_{\bar{z}} = \left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{z1}(k_x,k_y)\,\overline{EXP}\,dk_x\,dk_y\right\} EXP$$

$$E_{\bar{x}} = \frac{\eta}{k}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [k_y F_{z1}(k_x,k_y) + k_z G_{y1}(k_x,k_y)]\,\overline{EXP}\,dk_x\,dk_y\right.$$

$$\left. - \exp(-jk\bar{x}\sin\theta')[k\cos\theta'\exp(jk\bar{z}\cos\theta') - kR_{\|}\cos\theta'\exp(-jk\bar{z}\cos\theta')]\right\} EXP$$

$$E_{\bar{y}} = -\frac{\eta}{k}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [k_z D_{x1}(k_x,k_y) + k_x F_{z1}(k_x,k_y)]\,\overline{EXP}\,dk_x\,dk_y\right\} EXP$$

$$E_{\bar{z}} = \frac{\eta}{k}\left\{-k\sin\theta\exp(-jk\bar{x}\sin\theta')\,[\exp(jk\bar{z}\cos\theta') + R_{\|}\exp(-jk\bar{z}\cos\theta')]\right.$$

$$\left. + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [k_x G_{y1}(k_x,k_y) - k_y D_{x1}(k_x,k_y)]\,\overline{EXP}\,dk_x\,dk_y\right\} EXP$$

$$\overline{EXP} = \exp[+jk_x\bar{x} + jk_y\bar{y} - jk_z\bar{z}]$$

$$EXP = \frac{1}{\eta} \exp\left[-jk(x \sin\theta - Z \cos\theta)\right]$$

$$D_{x1}(k_x, k_y) = \frac{B_o a_{11} - A_o a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

$$G_{y1}(k_x, k_y) = \frac{A_o a_{22} - B_o a_{12}}{a_{11} a_{22} - a_{12} a_{21}}$$

$$F_{z1}(k_x, k_y) = \frac{k_y G_{y1}(k_x, k_y) + k_x D_{x1}(k_x, k_y)}{k_z}$$

$$A_o = -\eta k^2 k_y^2 k_z Q_1 - \eta k_z k^2 k_z^{\prime 2} Q_1 - W_o k_z k_z' k k'^2 - V k_z k_z' k k'^2$$

$$B_o = k_z k_z' k k'^2 S_o - \eta k^2 k_x k_y k_z Q_1$$

$$S_o = jk_y S(k_x + k \sin\theta', k_y)\eta T_{\|} \sin\theta' (k^2 - k'^2)/2\pi k'^2$$

$$V = -jT_{\|}\eta k \sin^2\theta' (k^2 - k'^2) S(k_x + k \sin\theta', k_y)/2\pi k'^2$$

$$W_o = j(k_x + k \sin\theta)\, \eta T_{\|} \sin\theta' (k^2 - k'^2)\, S(k_x + k \sin\theta', k_y)/2\pi k'^2$$

$$Q_1 = j(k^2 - k'^2)(1 - R_{\|}) \cos\theta' S(k_x + k \sin\theta', k_y)/2\pi k$$

$$a_{11} = \eta k_y^2 k_z' k'^2 + \eta k_z^2 k_z' k'^2 + \eta k^2 k_y^2 k_z + \eta k^2 k_z'^2 k_z$$

$$a_{12} = \eta k_y k_z' k'^2 k_x + \eta k^2 k_y k_z k_x$$

$$a_{21} = \eta k'^2 k_x k_z' k_y + \eta k^2 k_x k_y k_z$$

$$a_{22} = \eta k'^2 k_z^2 k_z' + \eta k'^2 k_x^2 k_z' + \eta k^2 k_z k_z'^2 + \eta k^2 k_x^2 k_z ,$$

where $R_{\|}$ is the local Fresnel reflection coefficient for a vertically polarized wave. The quantity $T_{\|}$ is equal to $1 + R_{\|}$. The form of $R_{\|}$ is as follows:

$$R_{\|} = \frac{k'^2 \cos\theta' - k\sqrt{k'^2 - k^2 \sin^2\theta'}}{k'^2 \cos\theta' + k\sqrt{k'^2 - k^2 \sin^2\theta'}}$$

$$\sin\theta' \approx \sin\theta - Z_x \cos\theta$$

$$\cos\theta' \approx \cos\theta + Z_x \sin\theta \ .$$

In order to obtain a workable expression for $R_{\|}$, it will be necessary to approximate $R_{\|}$ by the first two terms of its Taylor series expansion in Z_x, about $Z_x = 0$. When this is done, $R_{\|}$ becomes

$$R_{\|} \approx r_o + r_o' Z_x$$

$$r_o = \frac{k'^2 \cos\theta - k\sqrt{k'^2 - k^2 \sin^2\theta}}{k'^2 \cos\theta + k\sqrt{k'^2 - k^2 \sin^2\theta}}$$

$$r_o' = \frac{2k'^2 k \sin\theta\,(k'^2 - k^2)}{\sqrt{k'^2 - k^2 \sin^2\theta}\,\left\{k'^2 \cos\theta + k\sqrt{k'^2 - k^2 \sin^2\theta}\right\}^2}$$

At the surface where the fields are to be computed, $\overline{x} = 0$, $\overline{y} = 0$, and $\overline{z} = s(x,y)$. It is also necessary to write D_{x1}, G_{y1}, and F_{z1} in terms of slopes so as to make a manageable solution later on:

$$D_{x1}(k_x, k_y) = jc_1(d_1 + d_2 Z_x)\, S(k_x + k\sin\theta', k_y)$$

$$G_{y1}(k_x, k_y) = jc_1(g_1 + g_2 Z_x)\, S(k_x + k\sin\theta', k_y)$$

$$F_{z1}(k_x, k_y) = jc_1(f_1 + f_2 Z_x)\, S(k_x + k\sin\theta', k_y)$$

$$c_1 = [a_{11} a_{22} - a_{12} a_{21}]^{-1}$$

$$d_1 = a_{11}\beta_2\beta_o - a_{21}\alpha_8$$

$$d_2 = a_{11}\beta_2\beta_1 - a_{21}\alpha_9$$

$$g_1 = \alpha_8 a_{22} - \beta_2\beta_o a_{12}$$

$$g_2 = \alpha_9 a_{22} - \beta_2\beta_1 a_{12}$$

$$f_1 = (k_y g_1 + k_x d_1)/k_z$$

$$f_2 = (k_y g_2 + k_x d_2)/k_z$$

$$\alpha_8 = (1 - r_o)(\alpha_3 + \alpha_1) + \alpha_5 (1 + r_o)(k_x \sin\theta + k \sin^2\theta) + \alpha_6 (1 + r_o)$$

$$\alpha_9 = -\alpha_1 r_o' + \alpha_2 (1 - r_o) - \alpha_3 r_o' + \alpha_4 (1 - r_o) - \alpha_5 (1 + r_o)(2k \sin\theta \cos\theta + k_x \cos\theta)$$

$$+ \alpha_5 r_o' (k_x \sin\theta + k \sin^2\theta) + \alpha_6 r_o' + \alpha_7 (1 + r_o)$$

$$\beta_o = k_z'(1 + r_o) \sin\theta - k_x (1 - r_o) \cos\theta$$

$$\beta_1 = k_z' r_o' \sin\theta - k_z'(1 + r_o) \cos\theta - k_x (1 - r_o) \sin\theta + k_x r_o' \cos\theta$$

$$\beta_2 = \frac{k_y \eta k_z k (k^2 - k'^2)}{2\pi}$$

$$\alpha_1 = -\eta k_y^2 k k_z (k^2 - k'^2) \cos\theta / 2\pi$$

$$\alpha_2 = -\eta k k_y^2 k_z (k^2 - k'^2) \sin\theta / 2\pi$$

$$\alpha_3 = -\eta k_z k k_z'^2 (k^2 - k'^2) \cos\theta / 2\pi$$

$$\alpha_4 = -\eta k_z k k_z'^2 (k^2 - k'^2) \sin\theta / 2\pi$$

$$\alpha_5 = -\eta k_z k_z' k (k^2 - k'^2)/2\pi$$

$$\alpha_6 = \eta k_z k_z' k^2 \sin^2\theta (k^2 - k'^2)/2\pi$$

$$\alpha_7 = -2\eta k_z k_z' k^2 \sin\theta \cos\theta (k^2 - k'^2)/2\pi \quad .$$

In deriving the above expressions, terms containing Z_x^2, Z_x^3 . . . . . etc., have been neglected. The electric and magnetic fields evaluated on the surface become:

$$H_{\bar{x}} = \left\{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D_{x1} \exp(-jk_z s) dk_x dk_y \right\} \mathrm{EXP}$$

$$H_{\bar{y}} = \left\{ \exp(jks\cos\theta) + (r_o + r_o' Z_x)\exp(-jks\cos\theta) + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_{y1}\exp(-jk_z s) \cdot \right.$$

$$\left. dk_x\, dk_y \right\} \mathrm{EXP}$$

$$H_{\bar{z}} = \left\{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{z1} \exp(-jk_z s) dk_x\, dk_y \right\} \mathrm{EXP}$$

$$E_{\bar{x}} = \frac{\eta}{k} \left\{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (k_y F_{z1} + k_z G_{y1}) \exp(-jk_z s) dk_x\, dk_y \right.$$

$$- kZ_x \sin\theta\, [\exp(jks\cos\theta) - (r_o + r_o' Z_x)\exp(-jks\cos\theta)]$$

$$\left. - k\cos\theta \exp(jks\cos\theta) + k\cos\theta\,(r_o + r_o' Z_x)\exp(-jks\cos\theta) \right\} \mathrm{EXP}$$

$$E_{\bar{y}} = -\frac{\eta}{k} \left\{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (k_z D_{x1} + k_x F_{z1}) \exp(-jk_z s)\, dk_x\, dk_y \right\} \mathrm{EXP}$$

$$E_{\bar{z}} = \frac{\eta}{k} \left\{ -k(\sin\theta - Z_x\cos\theta)[\exp(jks\cos\theta) + (r_o + r_o' Z_x)\exp(-jks\cos\theta)] \right.$$

$$\left. + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (k_x G_{y1} - k_y D_{x1}) \exp(-jk_z s)\, dk_x\, dk_y \right\} \mathrm{EXP}\ . \qquad (5)$$

The $(\vec{i}\cos\theta + \vec{k}\sin\theta)$ term in equation (2) will now form the like-polarized backscattered field when the incident wave is vertically polarized. Using this term from equation (2) and the above-defined surface fields, we can obtain an expression for the like-polarized backscattered field E_{VV}:

$$E_{VV} = -K \iint [a_1' + b_1' Z_x] \exp\left\{ -2jkx\sin\theta + 2jkz\cos\theta \right\} dy\, dx + I_{VV}$$

$$a_1' = 2r_o\cos\theta \qquad b_1' = 2r_o\sin\theta + 2r_o'\cos\theta$$

$$I_{VV} = K \iiiint \left\{ A_1 + B_1 Z_x + C_1 s_x + D_1 Z_y + E_1 s_y \right\} S \exp(jas)$$

$$\cdot \exp\left\{-2jkx \sin\theta + 2jkZ\cos\theta\right\} dk_x\, dk_y\, dy\, dx$$

$$S = S(k_x + k\sin\theta', k_y) \qquad a = k\cos\theta - k_z$$

$$A_1 = \frac{-jc_1}{k}\left\{k_y f_1 + k_z g_1 + g_1 k\cos\theta\right\}$$

$$B_1 = \frac{-jc_1}{k}\left\{k_y f_2 + k_z g_2 + g_2 k\cos\theta + g_1 k\sin\theta\right\}$$

$$C_1 = \frac{-jc_1}{k}\left\{k_x g_1 - k_y d_1 + g_1 k\sin\theta\right\}.$$

$$D_1 = \frac{jc_1}{k\sin\theta}\left\{(k_z d_1 + k_x f_1)\cos\theta + kd_1\right\}$$

$$E_1 = -jc_1\left\{f_1\cos\theta - d_1\sin\theta\right\} .$$

Use has been made of the derived forms of D_{x1}, G_{y1}, and F_{z1}. It can easily be seen that E_{VV} consists of two terms the first of which depends solely on the large undulations and represents the quasi-specular component. The second term I_{VV}, although it depends on the large undulations through two slope terms and a phase term, arises because of the slightly rough surface $s(x,y)$. The form of E_{VV} is now the same as that for E_{HH}, the only difference being in the definitions of the coefficients on the slope terms. An expression for $<E_{VV}E^*_{VV}>$ can then be written directly without further work:

$$<E_{VV}E^*_{VV}> = KK^*\left\{\frac{\pi a_1'^2 L^2 G}{K_1} + \frac{8\pi\sigma^2 L^2 G a_1' b_1' k^2 \sin\theta\cos\theta}{K_1^2}\right.$$

$$\left. + \frac{4\pi L^2 G b_1'^2 \sigma^2 k^2 \sin^2\theta}{K_1^2}\right\} \exp\left\{-k^2 L^2 G \sin^2\theta / K_1\right\} + <I_{VV} I^*_{VV}> .$$

The equation for $<I_{VV}I^*_{VV}>$ will be the same as that for $<I_{HH}I^*_{HH}>$ except that A, B, C, D, and E will be replaced by A_1, B_1, C_1, D_1, and E_1 respectively. Once again, it can be seen that $<I_{VV}I^*_{VV}>$ will disappear when $\sigma_1 = 0$ leaving only the quasi-specular term.

b. Depolarized Term. The depolarized scattering coefficient for a vertically polarized incident wave is σ^o_{VH} . The objective now is to obtain an expression for $\langle E_{VH} E^*_{VH} \rangle$. The $\hat{j}$ component of equation (2) will give a basic equation for E_{VH} in terms of the surface fields:

$$E_{VH} = K \iint \Big\{ E_{\bar{x}} Z_y \csc\theta \cos^2\theta - E_{\bar{y}} \cos\theta - E_{\bar{z}} s_y \cos\theta + \eta H_{\bar{x}}$$

$$+ \eta H_{\bar{y}} Z_y \csc\theta \cos\theta + \eta H_{\bar{z}} s_x + E_{\bar{x}} Z_y \sin\theta + s_y E_{\bar{x}} \sin\theta$$

$$- E_{\bar{y}} Z_x \sin\theta - s_x E_{\bar{y}} \sin\theta \Big\} \exp(-jk\vec{n}_1 \cdot \vec{r})\, dy\, dx \quad .$$

The previously stated surface fields used in the above expression for E_{VH} results in the following equation:

$$E_{VH} = K \iint \Big\{ a_2' Z_y + b_2' s_y \Big\} \exp[-2jkx \sin\theta + 2jkZ \cos\theta]\, dy\, dx + I_{VH} \ ,$$

where

$$a_2' = 2r_o \csc\theta \cos\theta \quad b_2' = 2r_o \sin\theta \cos\theta$$

$$I_{VH} = K \iiiint \Big\{ A_1' + B_1' Z_x + C_1' s_x + D_1' Z_y + E_1' s_y \Big\} S \exp(j\,a\,s)$$

$$\cdot \exp[-2jkx \sin\theta + 2jkZ \cos\theta]\, dk_x\, dk_y\, dy\, dx$$

$$A_1' = \frac{jc_1}{k} [k_z d_1 \cos\theta + k_x f_1 \cos\theta + d_1 k]$$

$$B_1' = \frac{jc_1}{k} [(k_z d_2 + k_x f_2) \cos\theta + d_2 k + (k_z d_1 + k_x f_1) \sin\theta]$$

$$C_1' = \frac{jc_1}{k} [(k_z d_1 + k_x f_1) \sin\theta + f_1 k]$$

$$D_1' = \frac{jc_1}{k \sin\theta} [k_y f_1 + k_z g_1 + k g_1 \cos\theta]$$

$$E_1' = \frac{jc_1}{k} [(k_y f_1 + k_z g_1) \sin\theta - (k_x g_1 - k_y d_1) \cos\theta] \ .$$

The depolarized expression consists of the sum of two terms, the first of which is dependent only upon the two y slope terms. The term I_{VH} is similar to expressions derived previously. The form of E_{VH} can be seen to be the same as that for E_{HV} so that an equation for $<E_{VH}E^*_{VH}>$ can be written down directly:

$$<E_{VH}E^*_{VH}> = \frac{2\pi KK^* b_2'^2 \sigma_1^2 K_1 L^2 Gl^2}{(K_1 l^2 + L^2 G)^2} \exp\left(\frac{-k^2 \sin^2\theta}{\frac{K_1}{L^2 G} + \frac{1}{l^2}}\right) + <I_{VH}I^*_{VH}> \ .$$

The equation for $<I_{VH}I^*_{VH}>$ is of the same form as $<I_{HH}I^*_{HH}>$ except that A, B, C, D, and E will be replaced with A_1', B_1', C_1', D_1' and E_1'. Again, the depolarized term disappears if $\sigma_1 = 0$, although the expression is still a function of the large undulation parameters.

6. **Consideration of Local Vertical Components.** When the incident fields were written in the form of local coordinate components, the effects of the $\vec{x} \cdot \vec{j}$ and $\vec{z} \cdot \vec{j}$ components were neglected. It is the purpose of this section of the report to calculate the changes that would occur in the resultant backscatter coefficient if the local vertical terms are not ignored. The changes needed for an incident wave with horizontal polarization will be worked first. For a horizontally polarized incident wave, the incident field was given earlier and is repeated here for convenience:

$$\vec{E}_i^l = [\vec{x}(\vec{x}\cdot\vec{j}) + \vec{y}(\vec{y}\cdot\vec{j}) + \vec{z}(\vec{z}\cdot\vec{j})] \exp\left\{-jk(\bar{x}\sin\theta' - \bar{z}\cos\theta')\right\} \text{EXP}$$

$$\text{EXP} = \exp[-jk(x\sin\theta - Z\cos\theta)]$$

$$\vec{x}\cdot\vec{j} = -Z_y D_o \cos\theta$$

$$\vec{y}\cdot\vec{j} = 1$$

$$\vec{z}\cdot\vec{y} = -Z_y$$

$$D_o = (\sin\theta - Z_x\cos\theta)^{-1} \ .$$

The amplitude of the local vertical component can be computed from Maxwell's equation written in local coordinates:

$$\frac{j\vec{\nabla}\times\vec{E}_i^l}{k} = \eta\vec{H}_i^l \ .$$

The problem is now to compute the amplitude of the $\vec{y}$ component of $\vec{H}_i^l$. This amplitude will then be multiplied by each of the surface field components determined for vertical polarization and then added to the surface fields already given by equations (4) for horizontal polarization in order to obtain the new expressions for the total field. The $\vec{y}$ component of $\vec{H}^{I}_{1}$ will be written as $H^{I}_{1\bar{y}}$ and is computed to be

$$H^{I}_{1\bar{y}} \approx \frac{Z_y \csc\theta}{\eta} \exp\left\{-jk(\bar{x}\sin\theta' - \bar{z}\cos\theta)\right\}$$

In this calculation the higher order slope terms have been ignored. The total fields on the surface can now be written in the form:

$$E_{\bar{x}} = E_{1\bar{x}} + E_{2\bar{x}} \qquad H_{\bar{x}} = H_{1\bar{x}} + H_{2\bar{x}}$$

$$E_{\bar{y}} = E_{1\bar{y}} + E_{2\bar{y}} \qquad H_{\bar{y}} = H_{1\bar{y}} + H_{2\bar{y}}$$

$$E_{\bar{z}} = E_{1\bar{z}} + E_{2\bar{z}} \qquad H_{\bar{z}} = H_{1\bar{z}} + H_{2\bar{z}}$$

The subscript 1 refers to the surface field determined by a local incident horizontally polarized wave, while the subscript 2 refers to the surface fields determined by a local incident vertically polarized wave. The surface fields due to a local incident vertically polarized wave can be determined by multiplying $Z_y \csc\theta/\eta$ by the fields calculated in Appendix A and evaluating on the surface:

$$E_{2\bar{x}} = \frac{Z_y \csc\theta}{k}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (k_y F_{z1} + k_z G_{y1}) \exp(-jk_z s)\,dk_x\,dk_y \right.$$

$$- kZ_x \sin\theta\,[\exp(jks\cos\theta) - (r_o + r_o' Z_x)\exp(-jks\cos\theta)]$$

$$\left. - k\cos\theta \exp(jks\cos\theta) + k\cos\theta(r_o + r_o' Z_x)\exp(-jks\cos\theta)\right\} \text{EXP}$$

$$E_{2\bar{y}} = \frac{-Z_y \csc\theta}{k}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (k_z D_{x1} + k_x F_{z1}) \exp(-jk_z s)\,dk_x\,dk_y\right\} \text{EXP}$$

$$E_{2\bar{z}} = \frac{Z_y \csc\theta}{k}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (k_x G_{y1} - k_y D_{x1}) \exp(-jk_z s)\,dk_x\,dk_y \right.$$

$$\left. - k(\sin\theta - Z_x\cos\theta)\,[\exp(jks\cos\theta) + (r_o + r_o' Z_x)\exp(-jks\cos\theta)]\right\} \text{EXP}$$

$$\eta H_{2\bar{x}} = Z_y \csc\theta\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D_{x1} \exp(-jk_z s)\,dk_x\,dk_y\right\} \text{EXP}$$

$$\eta H_{2\bar{y}} = Z_y \csc\theta \left\{ \exp(jks\cos\theta) + (r_o + r_o' Z_x)\exp(-jks\cos\theta) + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_{y1} \exp(-jk_z s)\, dk_x\, dk_y \right\} \mathrm{EXP}$$

$$\eta H_{2\bar{z}} = Z_y \csc\theta \left\{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{z1} \exp(-jk_z s)\, dk_x\, dk_y \right\} \mathrm{EXP} \quad .$$

The above fields must be added to the fields given by equations (4) in order to obtain the total surface fields. The above equations are correction terms owing to the fact that local vertical polarization components exist. When the total surface fields are now substituted into the expression for E_{HH}, it is found that the quasi-specular term remains unchanged if high-order slope terms are still ignored. The only change in I_{HH} comes in a new definition of the D term. It can be shown that D now becomes

$$D = \csc\theta \cos^2\theta\, k_y k_x Q(1 + R_o) + \frac{\cos\theta \csc\theta}{k}(k_z d_1 + k_x f_1)c_1 j + jc_1 d_1 \csc\theta + \csc\theta \cos\theta (1 + R_o)\frac{k_y k_x}{k}(k_z - k_z')Q + \sin\theta (1 + R_o) k_y k_x Q \quad .$$

Two more terms have been added to D. When the new total surface fields are placed in the expression for E_{HV}, the definition of a_2 changes and the definition of D' changes as follows:

$$D' = -\frac{\csc\theta}{k}(k_y f_1 + k_z g_1) jc_1 - \csc\theta \cos\theta\, jc_1 g_1 + \csc\theta \cos\theta (1 + R_o)(k_x^2 + k_z k_z')Q + (1 + R_o)\left[\frac{k_z (k_x^2 + k_z k_z')}{k} + \frac{k_y^2 k_z'}{k}\right] Q \csc\theta$$

$$a_2 = 2 \csc\theta \cos\theta (r_o + R_o) \quad .$$

The term a_2 is not used in the final results and, therefore, is not needed. The new definition of the D' term has two more terms added which come from local vertical polarization terms.

A similar analysis to the above can be performed for the case of vertical polarization. This time, the interest lies in determining the amplitude of the local horizontal component. This can be done by using the following Maxwell equation:

$$\vec{\nabla} \times \vec{H}_1^I = j\omega\epsilon_o \vec{E}_1^I .$$

In the above equation, $\vec{H}_1^I$ is the incident magnetic field written in local coordinates for a vertically polarized incident wave. The equation for the $\bar{y}$ component of $\vec{E}_1^I$, $E^I_{1\bar{y}}$, is then

$$E^I_{\bar{y}} = -Z_y \csc\theta \exp\left\{-jk(\bar{x}\sin\theta' - \bar{z}\cos\theta')\right\} \text{EXP}$$

$$\text{EXP} = \exp\left\{-jk(x\sin\theta - Z\cos\theta)\right\} .$$

The equations for the surface fields for horizontal polarization must be multiplied by the amplitude $-Z_y \csc\theta$ and added to the surface fields given by equations (5) to obtain the total surface fields. When this is done, the total surface fields are then placed in the expressions for E_{VV} and E_{VH} to determine differences. With these changes, the definition of the D_1 term which is the coefficient of Z_y in I_{VV} becomes

$$D_1 = jc_1 \left[\frac{\cos\theta\csc\theta}{k}(d_1k_z + f_1k_x) + d_1\sin\theta + d_1\cos^2\theta\csc\theta\right]$$

$$+ \csc\theta(1+R_o)k_yk_xQ + \csc\theta\cos\theta(1+R_o)\frac{k_xk_y}{k}(k_z - k_z')Q .$$

The term D_1 is the only change that occurs in E_{VV}. The definition of two terms changes in E_{VH}. The terms are a_2' and D_1':

$$a_2' = 2\csc\theta\cos\theta(r_e + R_o)$$

$$D_1' = \frac{jc_1}{k}[k_yf_1\sin\theta + k_zg_1\sin\theta + g_1k\csc\theta\cos\theta + k_yf_1\csc\theta\cos^2\theta$$

$$+ k_zg_1\csc\theta\cos^2\theta] - \csc\theta\cos\theta(1+R_o)(k_x^2 + k_zk_z')Q$$

$$- \csc\theta(1+R_o)\left[\frac{k_z(k_x^2 + k_zk_z')}{k} + \frac{k_y^2k_z'}{k}\right]Q .$$

The only effective difference in E_{VH} is in the D_1' term since a_2' does not appear in the final result.

This ends the analysis portion of the report, the next section discusses the results in terms of graphs of the backscatter coefficient versus incidence angle.

III. DISCUSSION

7. **Evaluation of Results.** The purpose of this section of the report is to describe numerical calculations and to show and discuss the resultant graphs of scattering coefficient versus incidence angle. These graphs represent the final result of all derivations and are plotted for different surfaces, different soils, different moisture contents, and two different frequencies. The equations for $\sigma^o_{HH}, \sigma^o_{HV}, \sigma^o_{VV}$, and σ^o_{VH} involve double integrals in k_x and k_y which cannot be solved easily by analytical techniques. It is necessary, therefore, to use a numerical integration routine in order to obtain final calculations. A two-dimensional Simpson's rule was used to evaluate the double integrals. The question of limits of integration for the computer was determined by using the exponent term in $W(k_x + k\sin\theta, k_y)$. This exponent will decrease rapidly as k_x and k_y increase. If e^{-10} is assumed to be insignificant, then the limits of integration (k_{xm}, k_{ym}) will be

$$k_{xm} = k_{ym} = \pm 2\sqrt{5}/l$$

with $\theta = 0^o$. A computer program was written in Fortran IV for the calculation of the backscatter coefficients. The program inputs are frequency, limits and number of intervals for integration, standard deviation of the large and small undulations, the correlation parameters, and dielectric constant of the surface. The output of the program is the four scattering coefficients in decibels for different angles of incidence. The functions needed for integration are formed in four separate function subprograms, while integration itself is performed in the main program. Careful examination of the equations for the scattering coefficients shows that they are not a function of the absolute values of σ or L but only their ratio. For all calculations, the value of σ read into the program was left at 1.0 and the value of L was varied. This is not the case, however, with the small surface perturbations as both σ_1 and l appear to have importance by themselves and so were varied individually in the program. It is not possible to evaluate the scattering coefficients at $\theta = 0^o$ due to the $\csc\theta$ in the DD^* terms of the double integral. The $\csc\theta$ term came about because of the assumptions that were made in the derivations and these assumptions are invalid at $\theta = 0^o$. Computations were started at $\theta = 10^o$ and were performed in 10^o increments up to $\theta = 80^o$. Calculations were also made for the case where $\sigma_1 = 0$ or when the surface is just a smoothly undulating one. In this case, only the like-polarized scattering coefficients have values as average power for the depolarized terms disappear. These graphs allow a determination to be made of the effect of the small surface perturbations on the resultant backscatter. Two different frequencies were used in the calculations: 9.375 GHz and 5.87 GHz. For each separate frequency used, it was important to determine the validity of the very rough surface condition by evaluating $K_1 = 4k^2\sigma^2\cos^2\theta$ and making sure that $K_1 >> 1$. If σ is held constant ($\sigma = 1$) and K_1 is evaluated at $\theta = 80^o$, then only the lower frequency need be calculated assuming it satisfies the condition $K_1 >> 1$:

$K_1 = 1823$ when $f = 5.87$ GHz, $\theta = 80^\circ$

It can be seen that the very rough surface condition is good for all calculations. When the frequency was changed, the parameters σ_1 and l were also changed so that $k\sigma_1$ and kl remained constant. The dielectric constants used were taken from Lundien[11] and represent soils with certain moisture contents and dry densities. The soil type, moisture content, and dry density of the soil are written on the curves along with the other data. The graphs which follow this section are labeled 1 through 30 with all appropriate data labeled on each one. Preceding the σ^0 curves is a curve of the correlation function $\rho(r)$. It can be seen that the correlation distance is larger than L. By far, the most important factor affecting the results, at least for like-polarized terms, is the ratio σ/L. The effects of polarization are noticed more at large angles of incidence than at small angles. This agrees very well with the experimental results discussed by Beckmann and Spizzichino.[12] This would indicate that if surface discrimination is wanted by polarization variation, then it possibly should be done at large angles of incidence.

The cause of depolarization is the slightly rough surface. Yet, if a slightly rough surface is taken by itself so that $\sigma = 0$; then no depolarization results for backscatter—at least, for first-order perturbation terms. Also, the type of depolarization that results from the large undulations by themselves ($\sigma_1 = 0$) depends on the sum of the two Fresnel reflection coefficients and does not appear in the final results. This is very interesting because the composite surface ends up with depolarized terms due to the fact that the local surface fields determined by the small perturbation technique are computed in a local coordinate system which is tilted with respect to the original coordinate system. This means that the depolarization results from a tilted, slightly rough surface. It can be seen from the graphs that the two depolarized terms are, in general, not equal over all angles. The cause of this could be either the result of the approximations made in the solution or there really is a difference in the two terms. It is not possible at present to determine which case is correct.

The effects of each individual parameter will be discussed briefly. The effect of differing moisture contents can be seen by examining curves 1 through 4 and curves 5 through 8. The higher moisture content results in a higher dielectric constant which yields a little higher value of σ^0 for each angle of incidence and for all four polarizations. Curves 9 through 12 are of a different soil type, but care must be taken in comparing these graphs with curves 1 through 4 because the moisture contents differ. If the difference in moisture contents could be taken out there probably would be very little difference in the two sets of graphs. The effect of changing the σ/L ratio can be

[11] J. R. Lundien, "Terrain Analysis by Electromagnetic Means," Technical Report No. 3-693, Waterways Experiment Station, Vicksburg, Miss.

[12] Beckmann and Spizzichino.

seen by comparing curves 1 through 4 and curves 13 through 16. This effect is particularly noticeable at intermediate angles for the like-polarized terms where the difference becomes very dramatic. The ratio σ/L can be seen to affect the like-polarized terms much more than the depolarized terms.

The influence of the correlation distance (l) of the slightly rough surface can be seen by comparing curves 1 through 4 with curves 17 through 20. Increasing the correlation distance l has the result of bringing the σ^0 curves down at large angles of incidence. For the like-polarized terms, this effect becomes noticeable for $\theta > 40°$. In the depolarized case, the whole curve is lowered; but the differences still become greater at larger incidence angles. The influence of σ_1, the standard deviation of the slightly rough surface, can be seen by comparing curves 1 through 4 with curves 21 through 24, and curves 25 and 26. This parameter has little effect on the like-polarized term until angles of incidence $\theta > 40°$ are reached. The effect of σ_1 on the depolarized terms covers the entire range of incidence angles, and the difference increases a little at large angles. All of the above comparisons have been made at the frequency of 9.375 GHz. It would not be reasonable to expect that exactly the same effects would occur at other frequencies. An idea of frequency differences can be obtained by comparing curves 1 through 4 with curves 27 through 30. For all four polarizations, the X-band frequency gives a slightly higher value of σ^0 for all angles of incidence.

IV. CONCLUSIONS

8. **General.** The following general conclusions can be made from the analysis and the graphs.

a. A correct theory for explaining radar backscatter must consider a composite surface as has been demonstrated in this paper.

b. Depolarization results from the effects of the tilted, slightly rough surface.

9. **Specific.** The following specific conclusions can be made from the plotted data at the frequency 9.375 GHz.

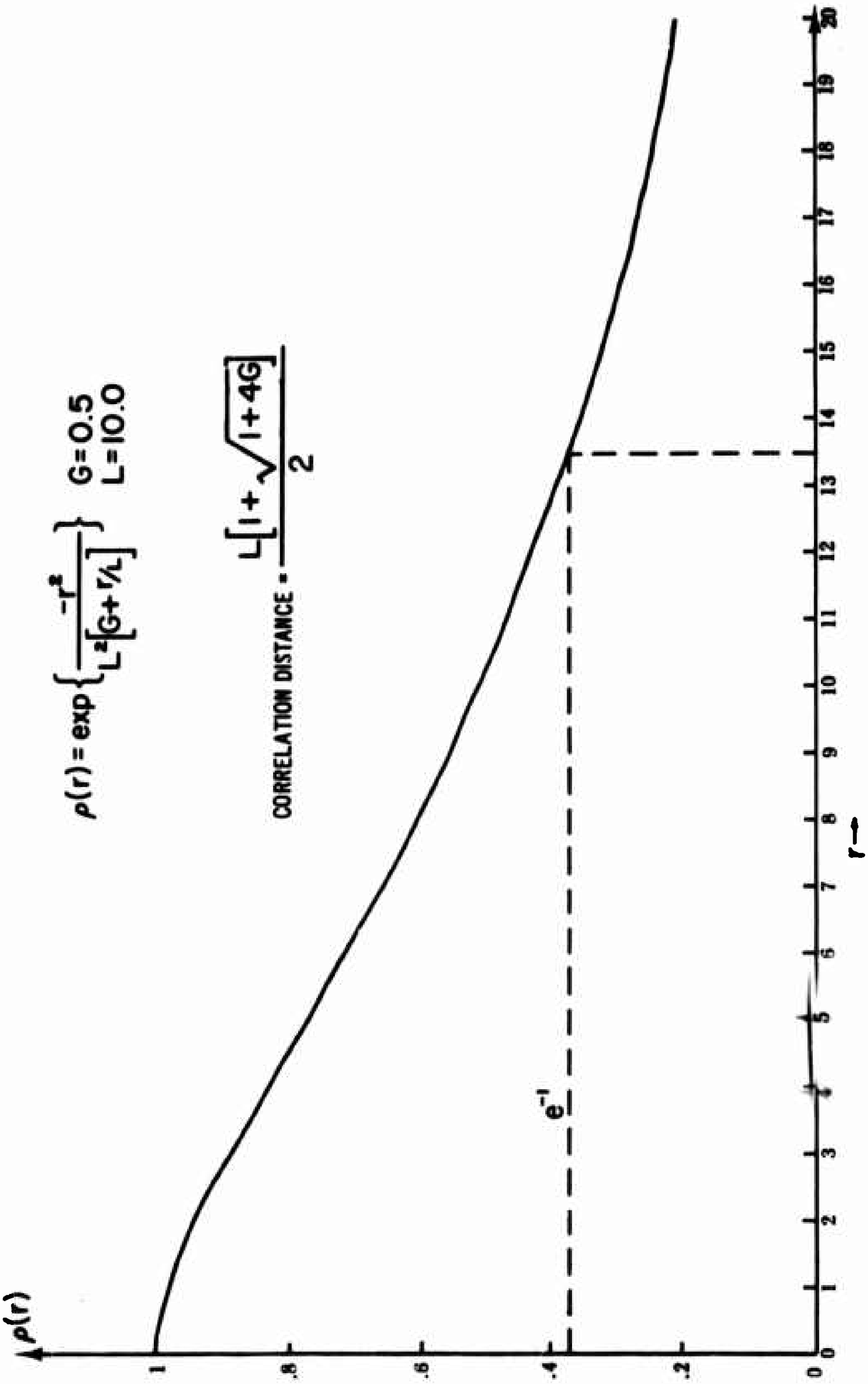
a. The ratio σ/L affects the like-polarized terms much more than the depolarized terms.

b. An increase in the moisture content of a soil increases σ^0 slightly for all angles of incidence but does not significantly change the shape of σ^0 versus θ curve.

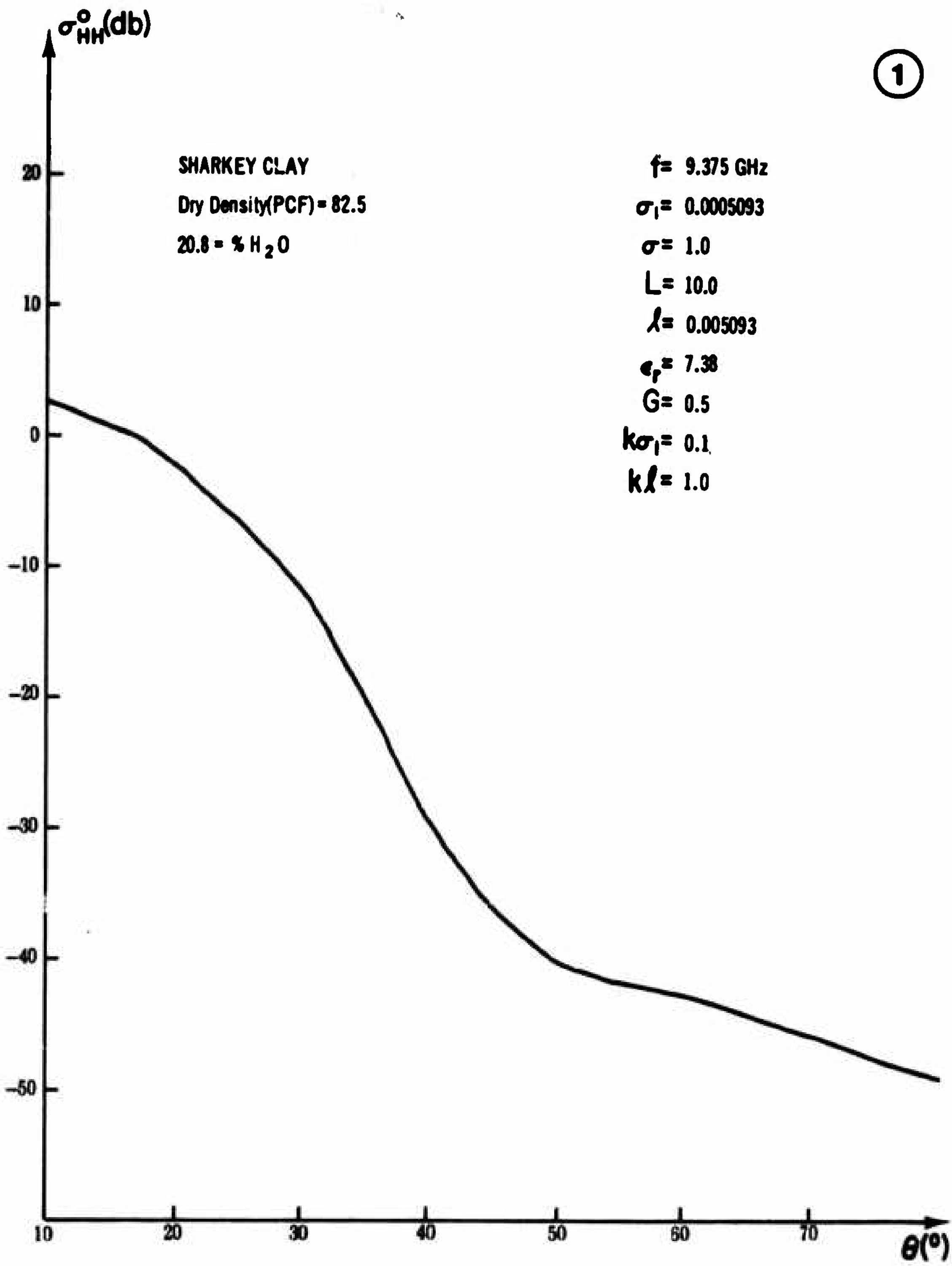
c. An increase in the correlation distance for the slightly rough surface results in σ° being less at high angles of incidence ($\theta > 40°$) for like-polarized term. For the depolarized terms, σ° is less for all angles of incidence.

d. An increase in σ_1, the standard deviation of the slightly rough surface undulations, results in a higher σ°. In the case of the like-polarized terms, a higher σ° becomes noticeable for $\theta > 40°$. In the case of the depolarized term, a higher σ° is noticeable for all angles of incidence.

e. It was shown from the graphs that the changes that occur in σ° due to a variation of parameter inputs is not large in many cases. This would indicate that if a radar is used for determining σ° for purposes of identification and discrimination then a very accurate calibration would be necessary.



$\rho(r) = \exp\left\{\frac{-r^2}{L^2\left[G + r/L\right]}\right\}$
G=0.5
L=10.0
CORRELATION DISTANCE = $\frac{L\left[1+\sqrt{1+4G}\right]}{2}$
$\rho(r)$
e^{-1}
1
.8
.6
.4
.2
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
r→



σ^o_{HH}(db)
1
SHARKEY CLAY
Dry Density(PCF) = 82.5
20.8 = % H_2O
f= 9.375 GHz
σ_l= 0.0005093
σ= 1.0
L= 10.0
ℓ= 0.005093
ϵ_r= 7.38
G= 0.5
$k\sigma_l$= 0.1
$k\ell$= 1.0
20
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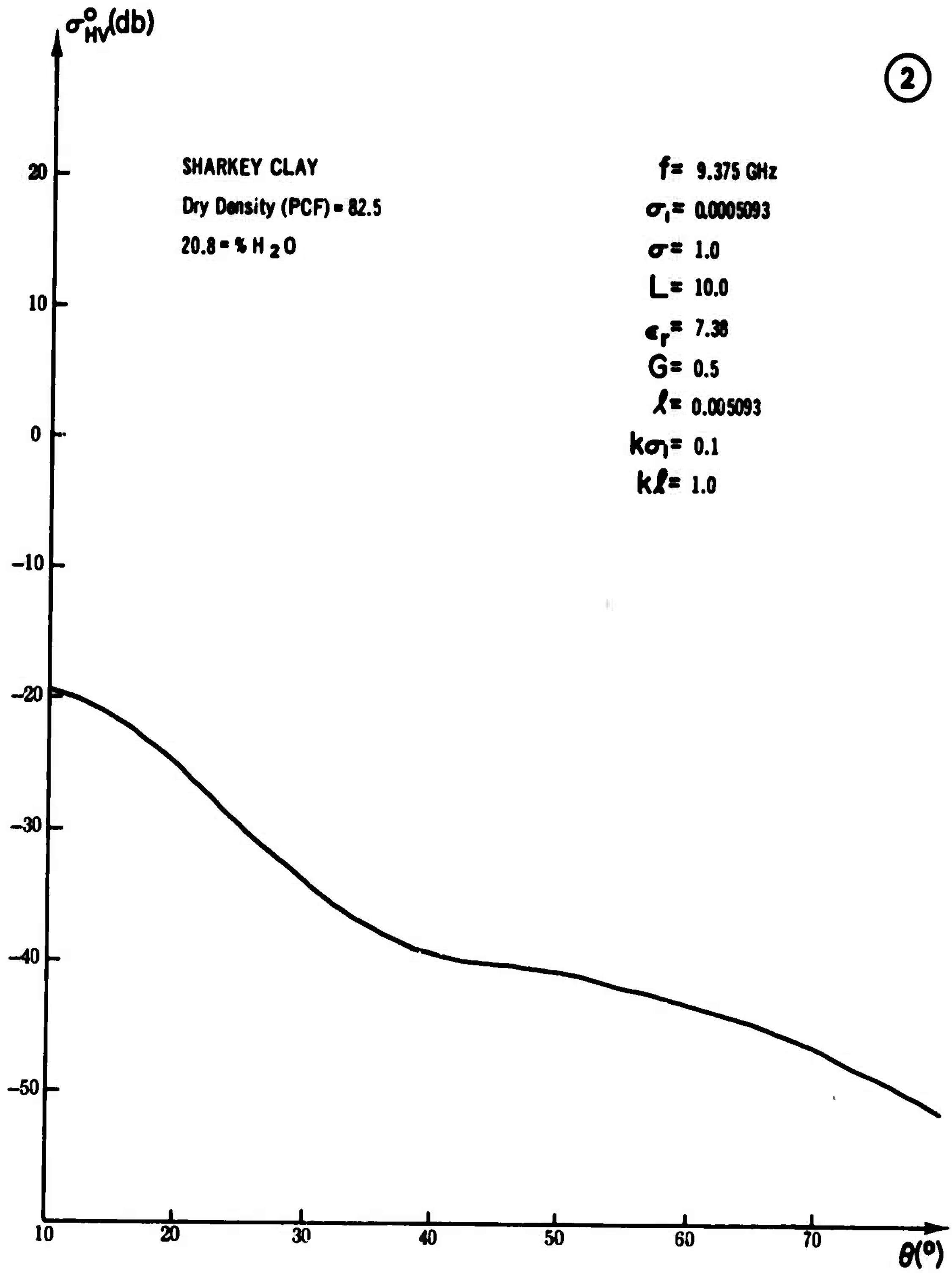
0
-10
-20
-30
-40
-50
10
20
30
40
50
60
70
θ(°)


σ^{o}_{HV}(db)
2
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f= 9.375 GHz
σ_1= 0.0005093
σ= 1.0
L= 10.0
ϵ_r= 7.38
G= 0.5
ℓ= 0.005093
$k\sigma_1$= 0.1
$k\ell$= 1.0
20
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60
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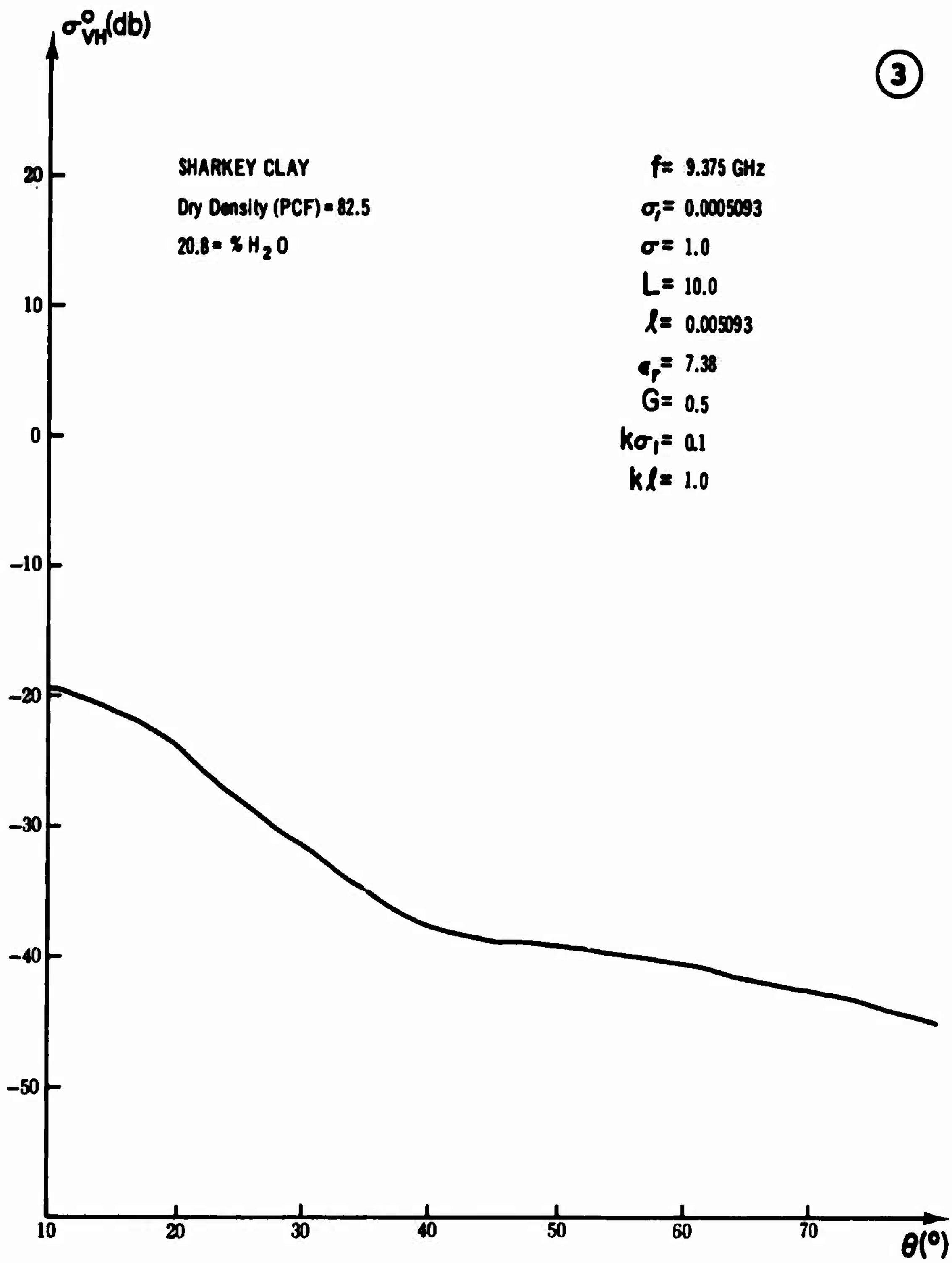
θ(°)


σ^o_{VH}(db)
3
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f= 9.375 GHz
σ_l= 0.0005093
σ= 1.0
L= 10.0
ℓ= 0.005093
ϵ_r= 7.38
G= 0.5
$k\sigma_l$= 0.1
$k\ell$= 1.0
20
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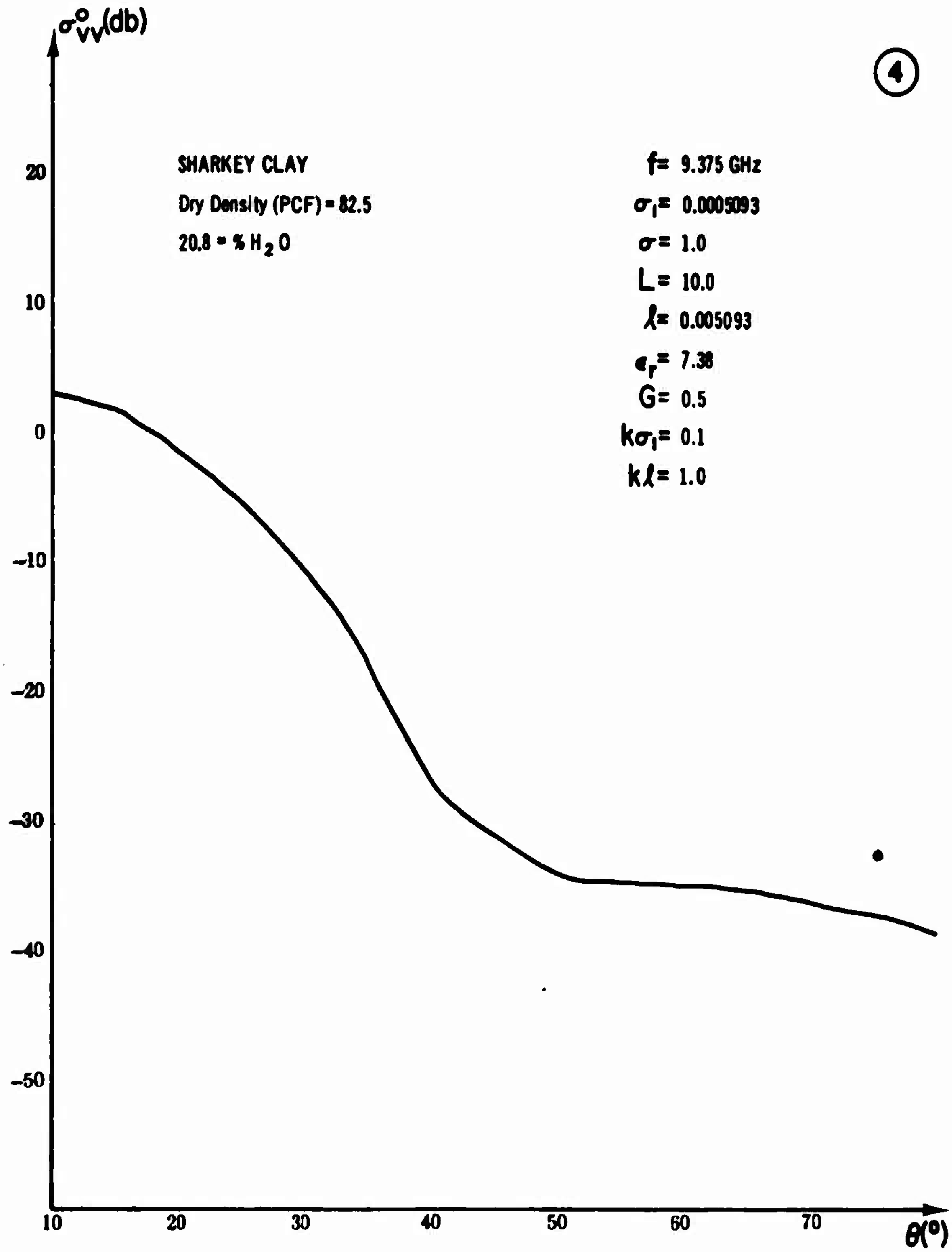
θ(°)



σ^o_{VV}(db)
4
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f = 9.375 GHz
σ_1 = 0.0005093
σ = 1.0
L = 10.0
ℓ = 0.005093
ϵ_r = 7.38
G = 0.5
$k\sigma_1$ = 0.1
$k\ell$ = 1.0
20
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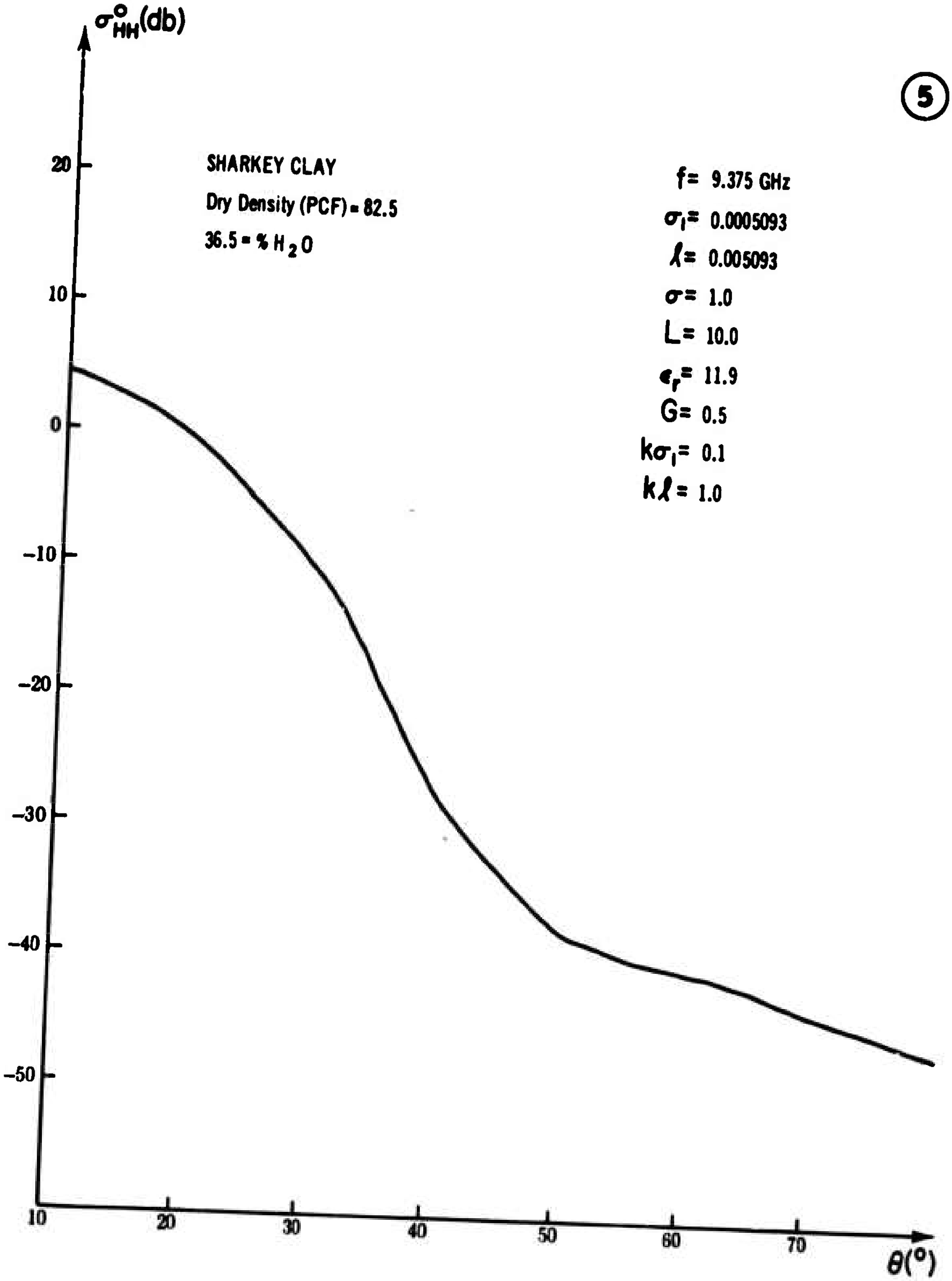
θ(°)


σ^o_{HH}(db)
5
SHARKEY CLAY
Dry Density (PCF) = 82.5
36.5 = % H_2O
f= 9.375 GHz
σ_I= 0.0005093
ℓ= 0.005093
σ= 1.0
L= 10.0
ϵ_r= 11.9
G= 0.5
$k\sigma_I$= 0.1
$k\ell$= 1.0
20
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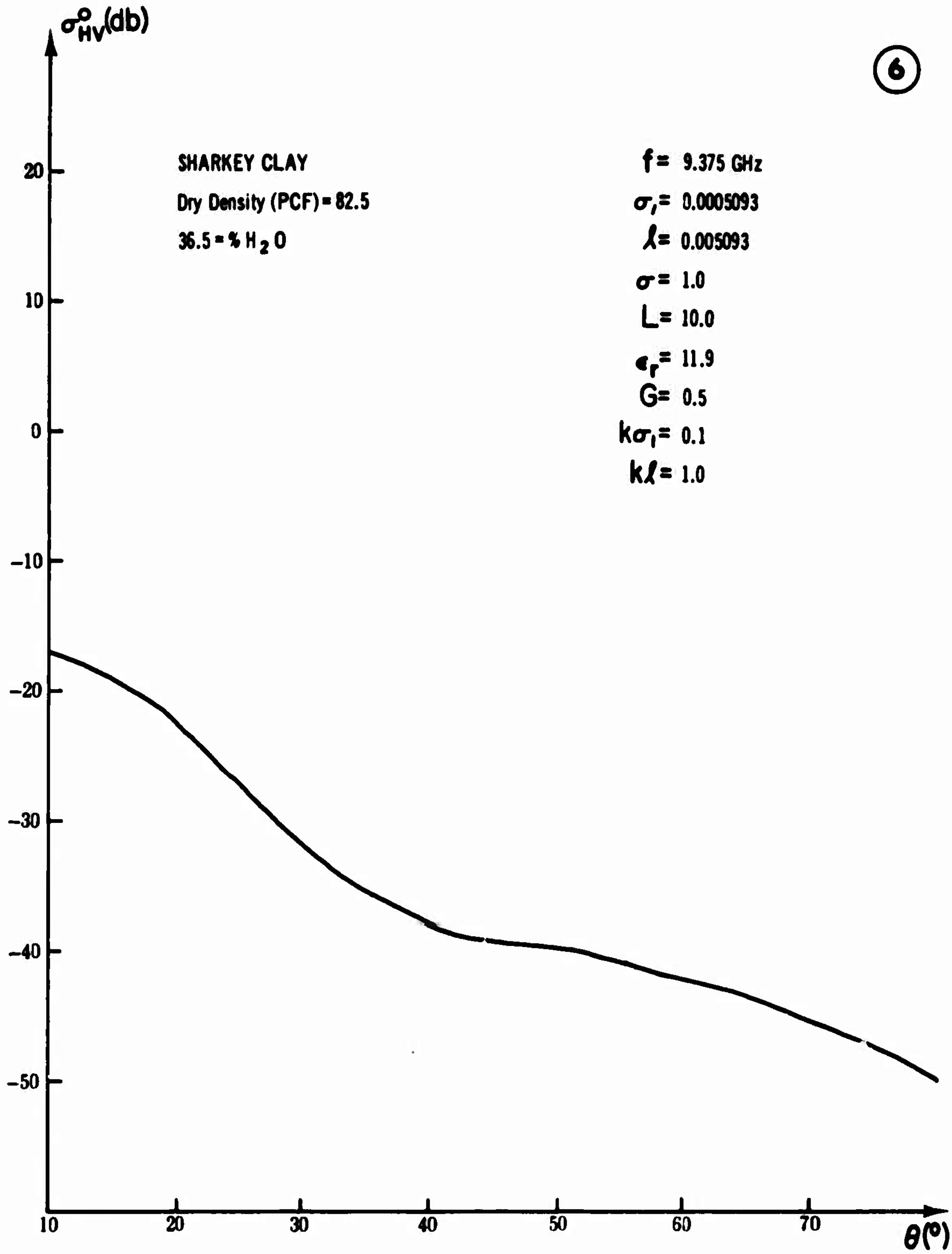
θ(°)


σ^o_{HV}(db)
6
SHARKEY CLAY
Dry Density (PCF) = 82.5
36.5 = % H_2O
f = 9.375 GHz
σ_l = 0.0005093
ℓ = 0.005093
σ = 1.0
L = 10.0
ϵ_r = 11.9
G = 0.5
$k\sigma_l$ = 0.1
$k\ell$ = 1.0
20
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-30
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-50
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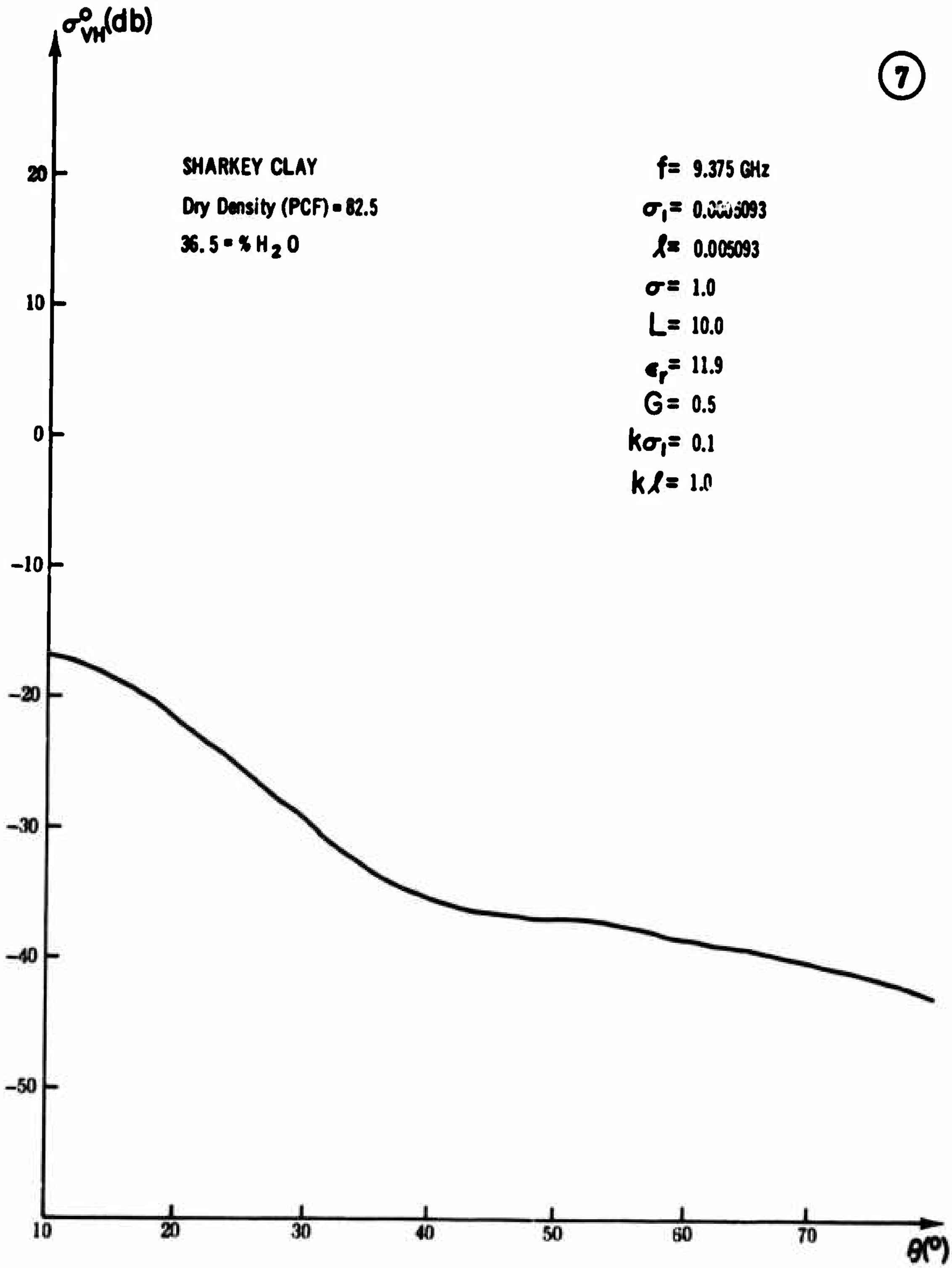
60
70
θ(°)



σ^o_{VH}(db)
7
SHARKEY CLAY
Dry Density (PCF) = 82.5
36.5 = % H_2O
f= 9.375 GHz
σ_1= 0.0005093
ℓ= 0.005093
σ= 1.0
L= 10.0
ϵ_r= 11.9
G= 0.5
$k\sigma_1$= 0.1
$k\ell$= 1.0
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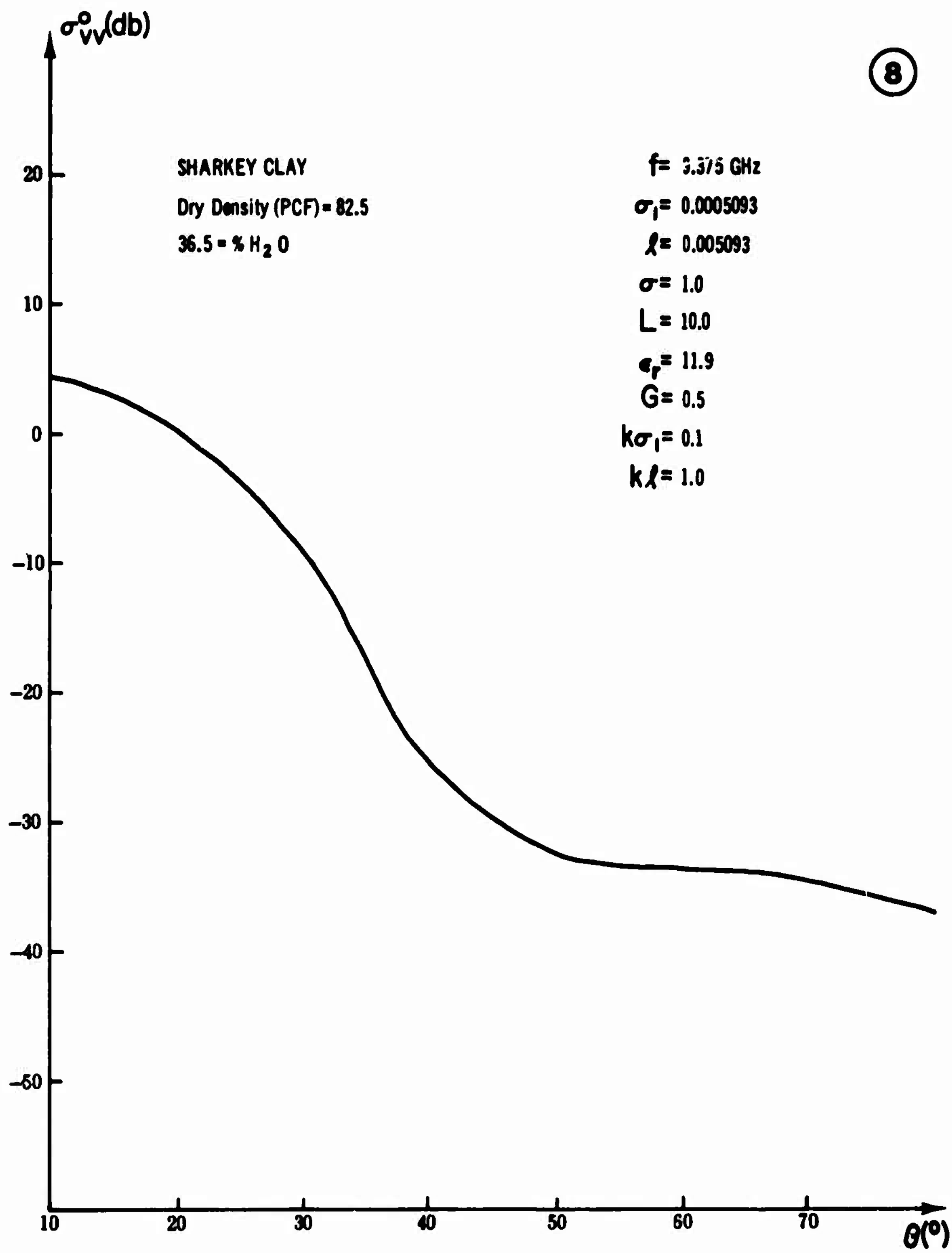
θ(°)



σ^o_{vv}(db)
8
SHARKEY CLAY
Dry Density (PCF) = 82.5
36.5 = % H_2O
f = 3.375 GHz
σ_l = 0.0005093
ℓ = 0.005093
σ = 1.0
L = 10.0
ϵ_r = 11.9
G = 0.5
$k\sigma_l$ = 0.1
$k\ell$ = 1.0
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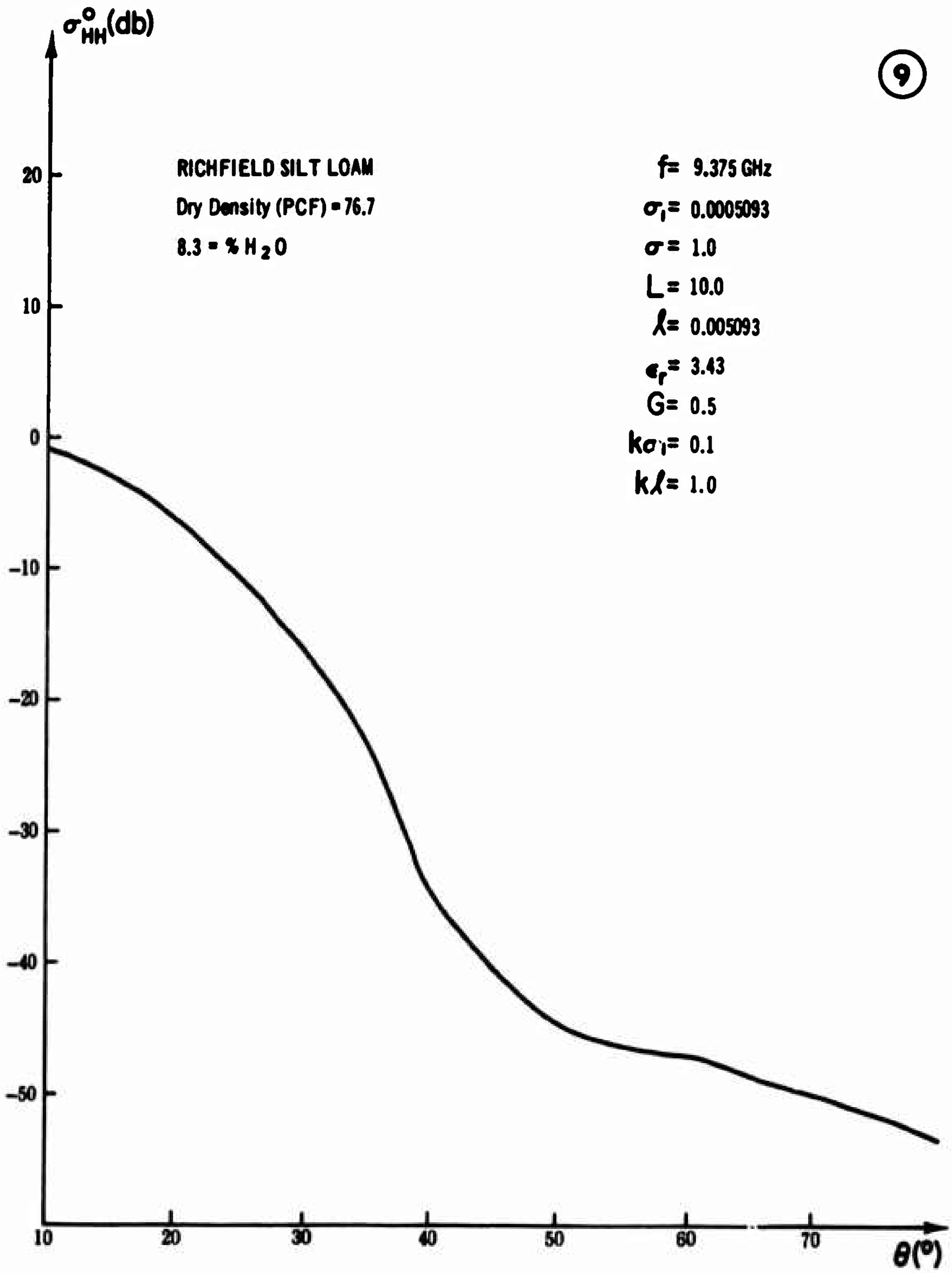
θ(°)



σ^o_{HH}(db)
9
20
10
0
-10
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-30
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-50
RICHFIELD SILT LOAM
Dry Density (PCF) = 76.7
8.3 = % H_2O
f= 9.375 GHz
σ_l= 0.0005093
σ= 1.0
L= 10.0
ℓ= 0.005093
ϵ_r= 3.43
G= 0.5
$k\sigma_l$= 0.1
$k\ell$= 1.0
10
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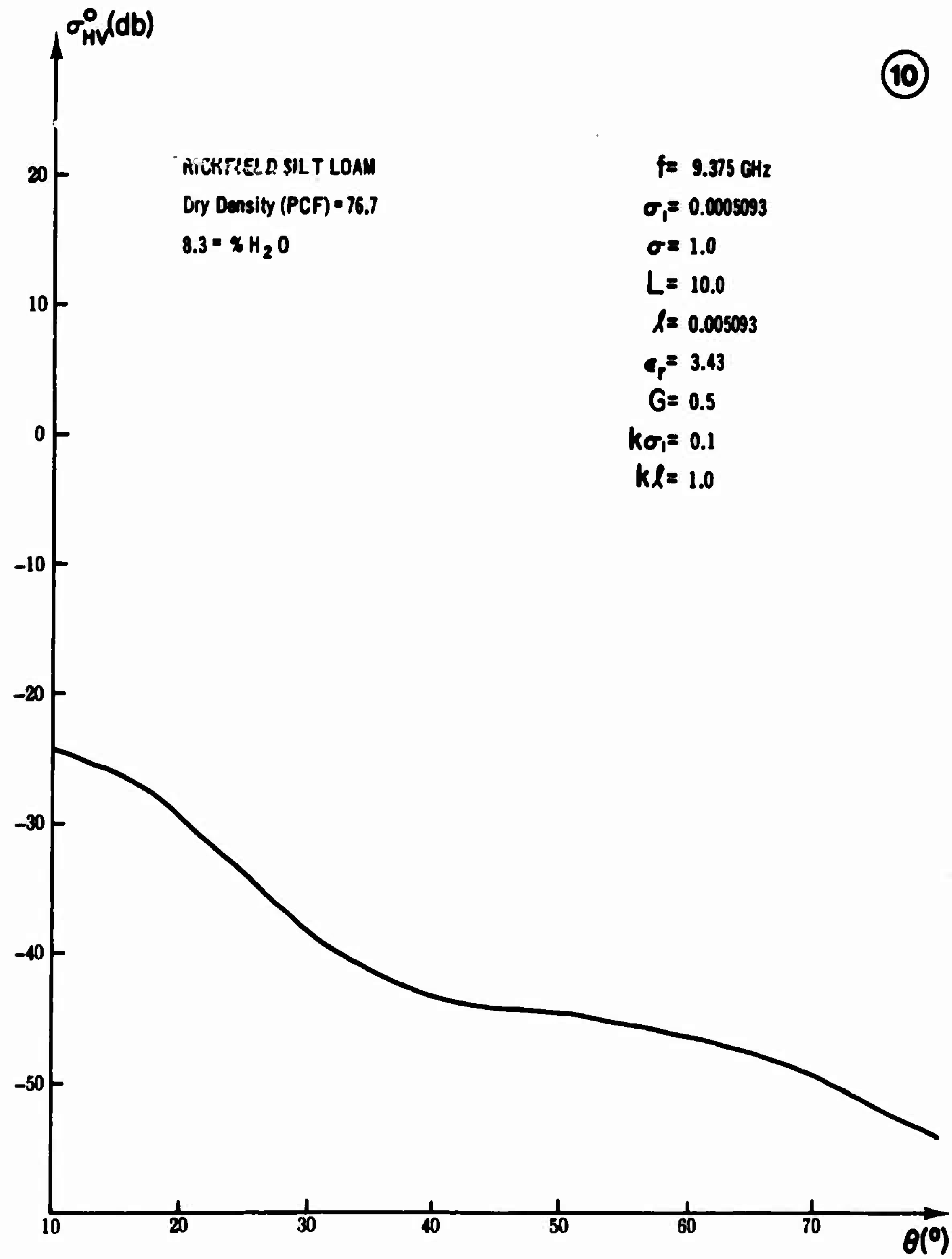
θ(°)



σ^o_{HV}(db)
10
RICKFIELD SILT LOAM
Dry Density (PCF) = 76.7
8.3 = % H_2O
f= 9.375 GHz
σ_1= 0.0005093
σ= 1.0
L= 10.0
ℓ= 0.005093
ϵ_r= 3.43
G= 0.5
$k\sigma_1$= 0.1
$k\ell$= 1.0
20
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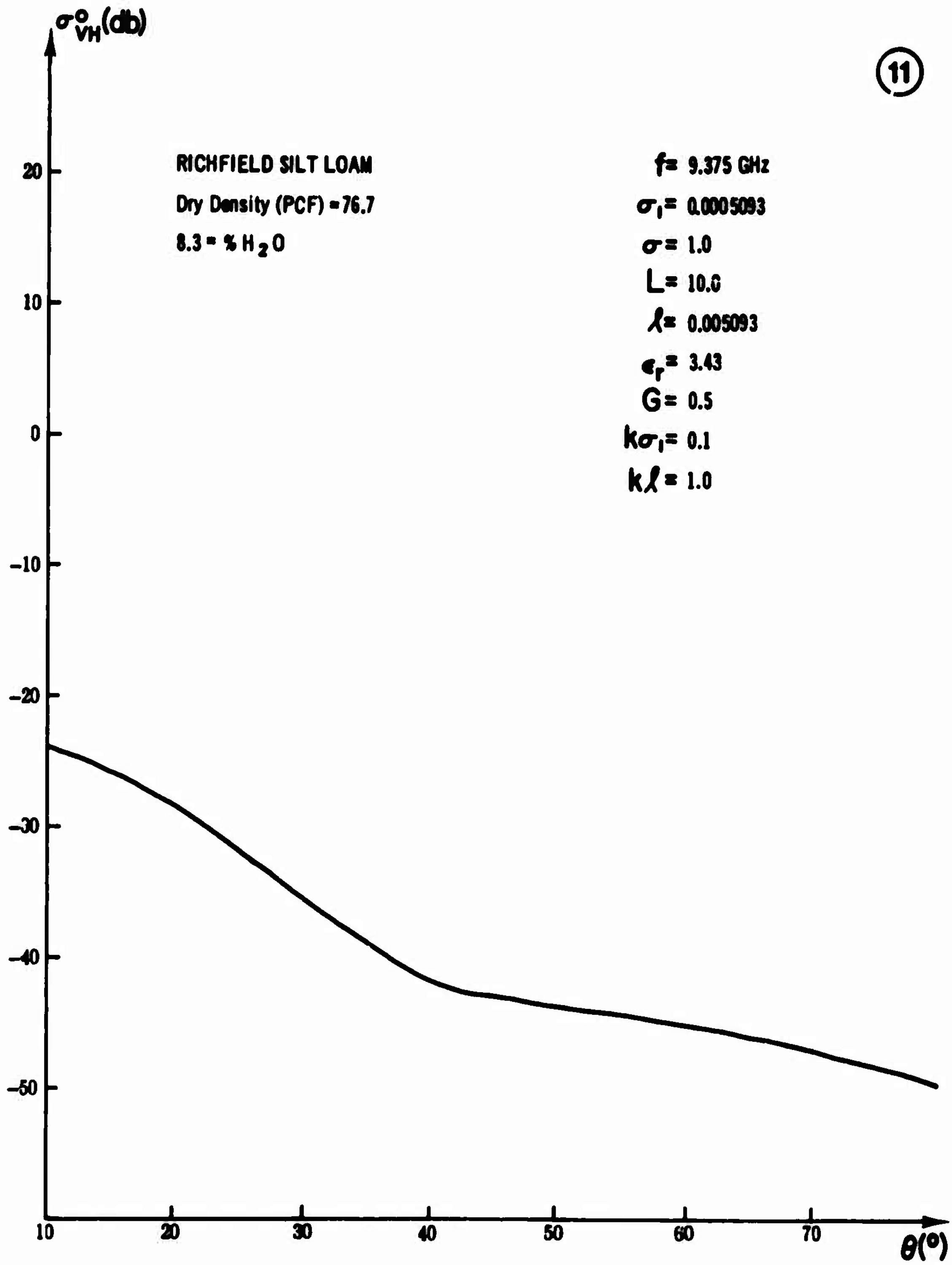
θ(°)



σ^o_{VH}(db)
11
RICHFIELD SILT LOAM
Dry Density (PCF) = 76.7
8.3 = % H_2O
f= 9.375 GHz
σ_l= 0.0005093
σ= 1.0
L= 10.0
ℓ= 0.005093
ϵ_r= 3.43
G= 0.5
$k\sigma_l$= 0.1
$k\ell$= 1.0
20
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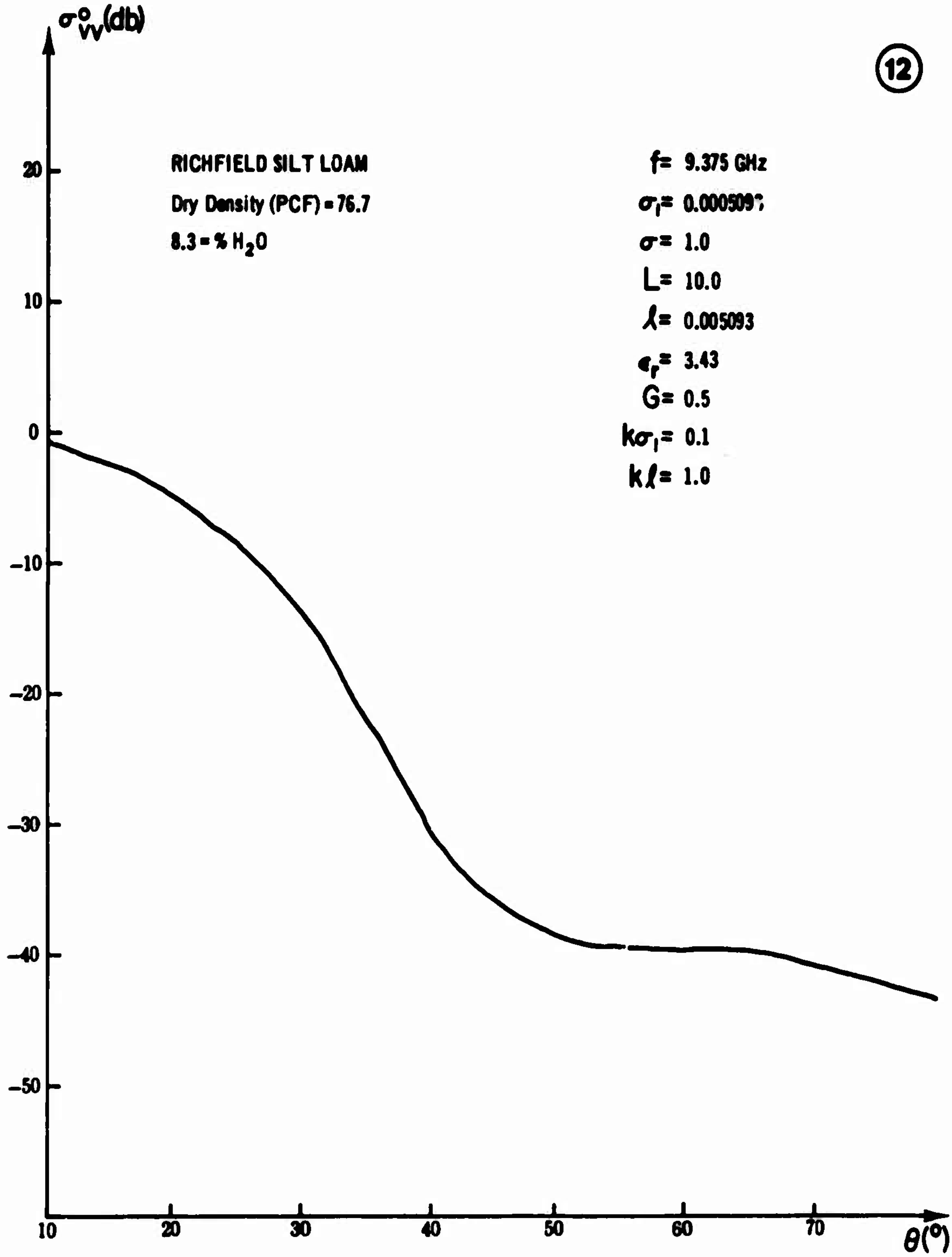
θ(°)



12
σ^o_{VV}(db)
RICHFIELD SILT LOAM
Dry Density (PCF) = 76.7
8.3 = % H_2O
f= 9.375 GHz
σ_l= 0.000509?
σ= 1.0
L= 10.0
ℓ= 0.005093
ϵ_r= 3.43
G= 0.5
$k\sigma_l$= 0.1
$k\ell$= 1.0
20
10
0
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10
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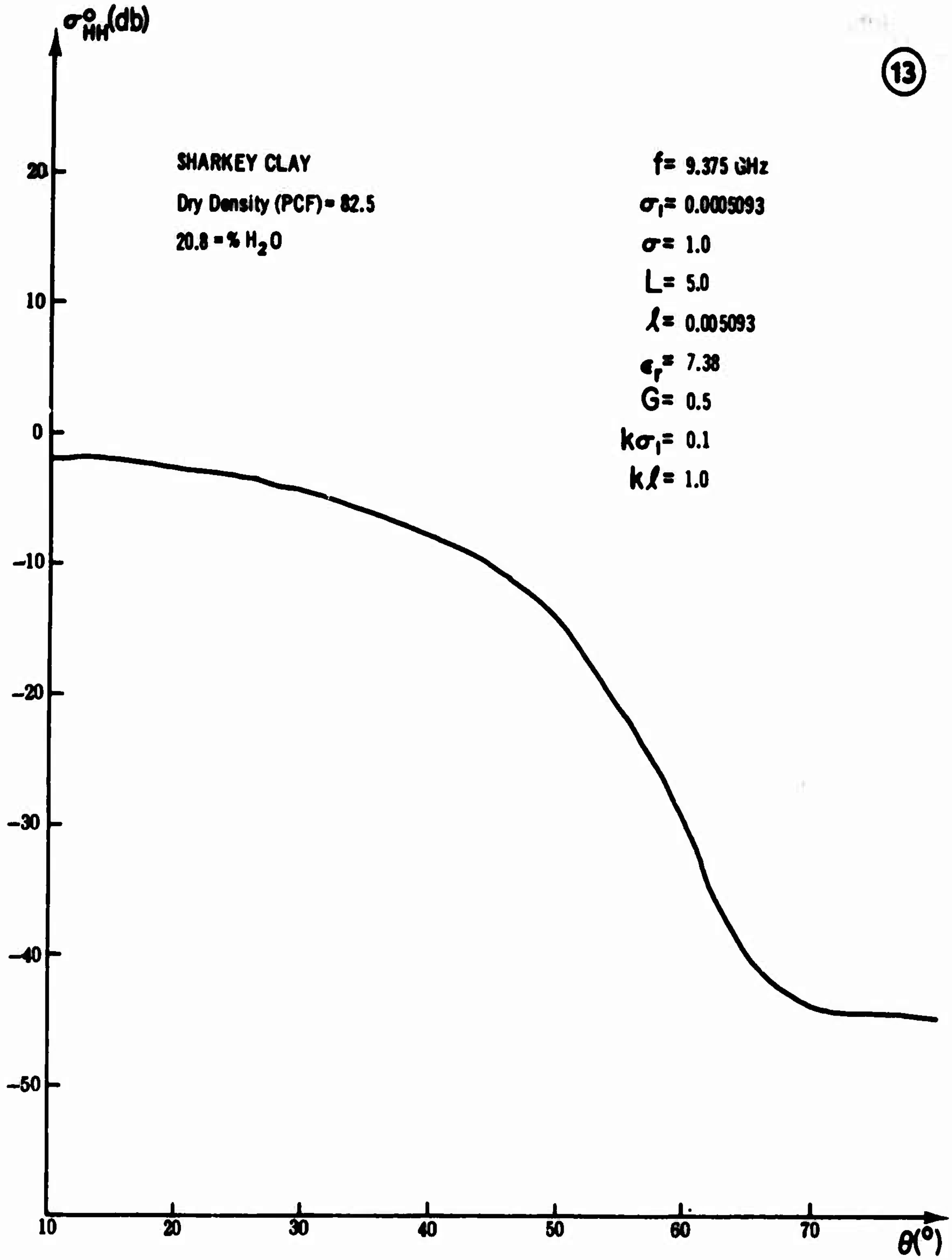
30
40
50
60
70
θ(°)



σ^o_{HH}(db)
13
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f = 9.375 GHz
σ_1 = 0.0005093
σ = 1.0
L = 5.0
ℓ = 0.005093
ϵ_r = 7.38
G = 0.5
$k\sigma_1$ = 0.1
$k\ell$ = 1.0
20
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40
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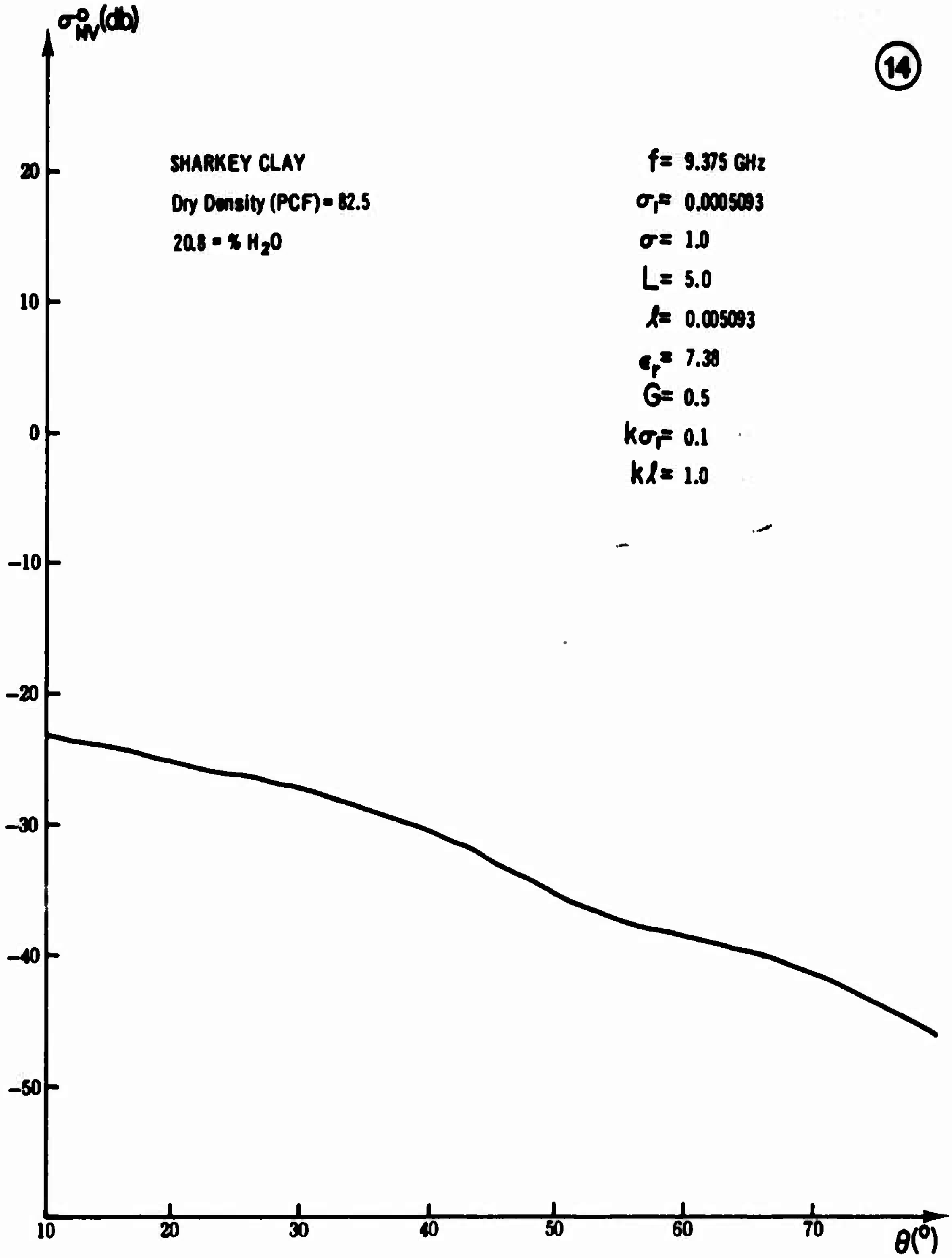
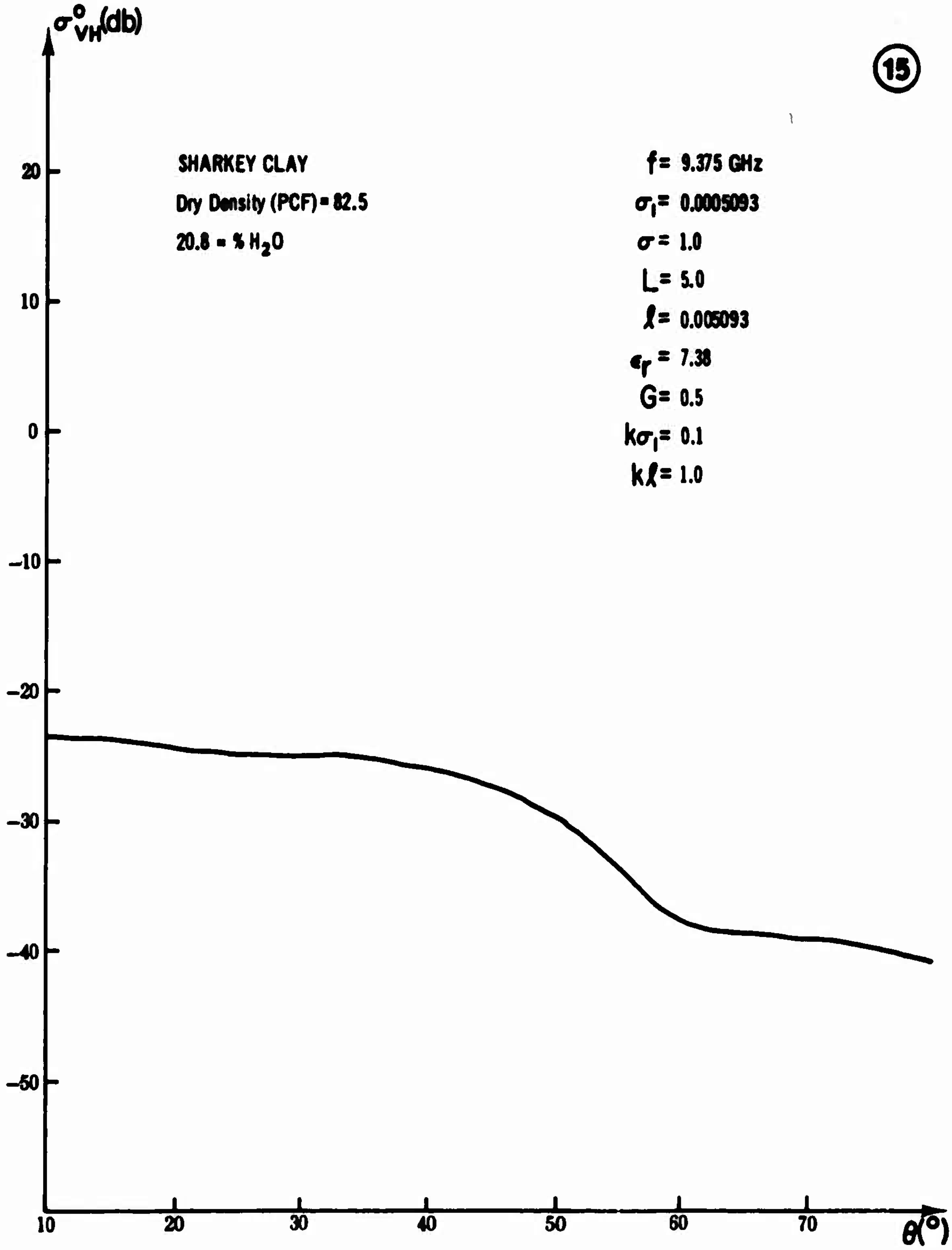
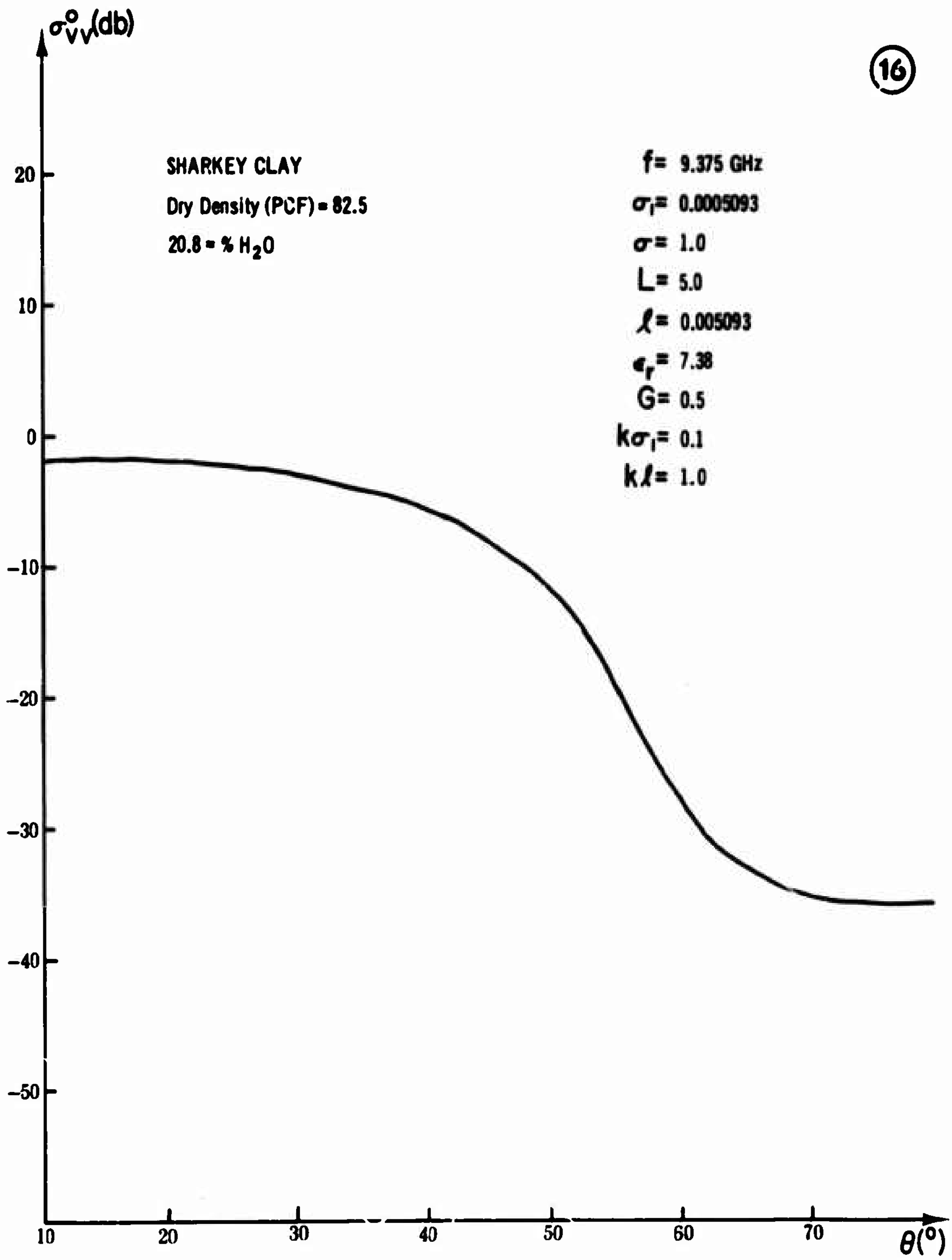
70
$\theta(^o)$



σ^o_{HV}(db)
14
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f= 9.375 GHz
σ_l= 0.0005093
σ= 1.0
L= 5.0
l= 0.005093
ϵ_r= 7.38
G= 0.5
$k\sigma_r$= 0.1
kl= 1.0
20
10
0
−10
−20
−30
−40
−50
10
20
30
40
50
60
70
θ(°)


σ^o_{VH}(db)
15
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f = 9.375 GHz
σ_l = 0.0005093
σ = 1.0
L = 5.0
ℓ = 0.005093
ϵ_r = 7.38
G = 0.5
$k\sigma_l$ = 0.1
$k\ell$ = 1.0
20
10
0
−10
−20
−30
−40
−50
10
20
30
40
50
60
70
θ(°)



σ^{o}_{VV}(db)
16
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f= 9.375 GHz
σ_1= 0.0005093
σ= 1.0
L= 5.0
ℓ= 0.005093
ϵ_r= 7.38
G= 0.5
$k\sigma_1$= 0.1
$k\ell$= 1.0
20
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−40
−50
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20
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40
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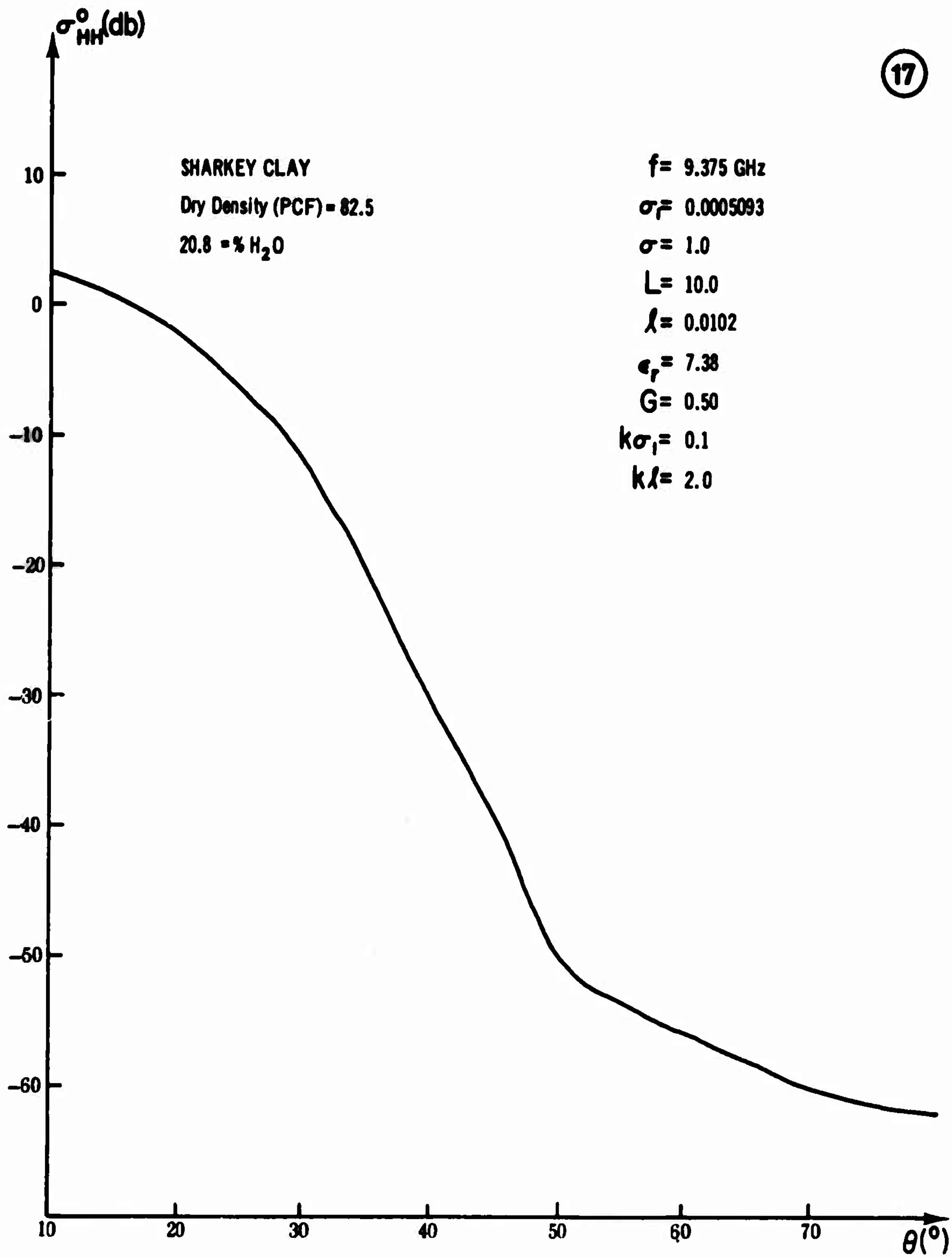
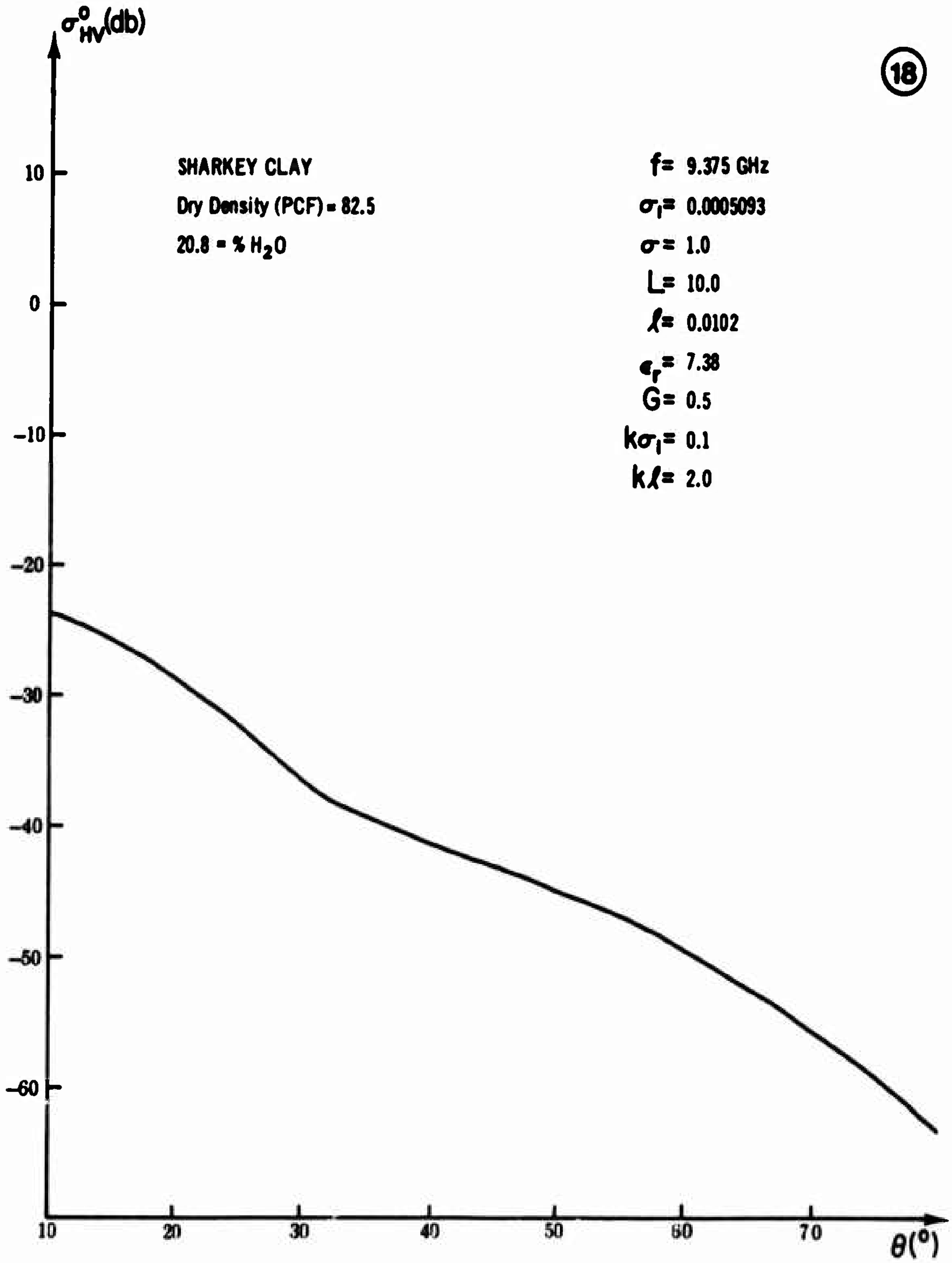
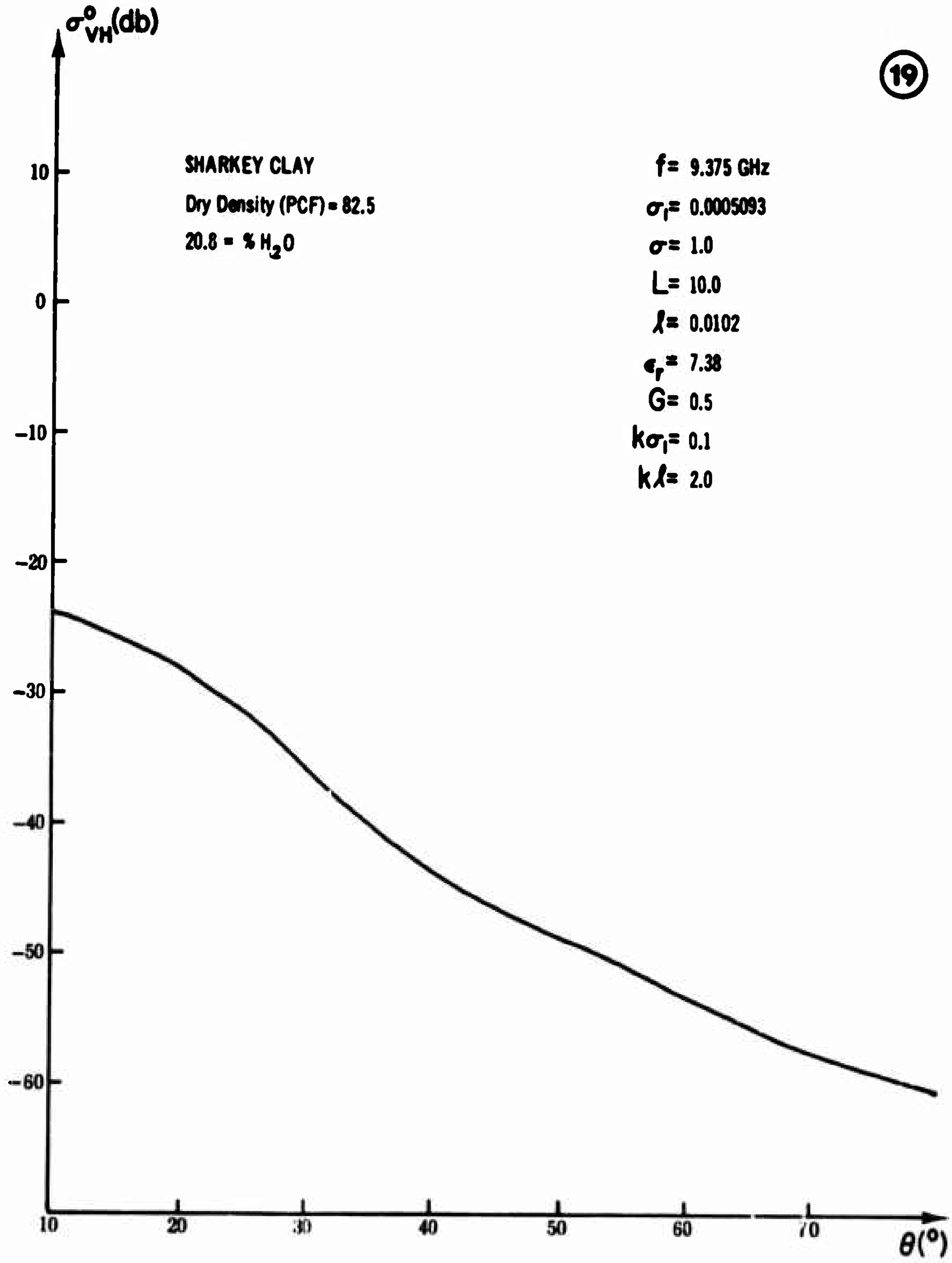
60
70
θ(°)



σ^o_{HH}(db)
17
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f = 9.375 GHz
σ_r = 0.0005093
σ = 1.0
L = 10.0
ℓ = 0.0102
ϵ_r = 7.38
G = 0.50
$k\sigma_1$ = 0.1
$k\ell$ = 2.0
10
0
-10
-20
-30
-40
-50
-60
10
20
30
40
50
60
70
θ(°)



σ^o_{HV}(db)
18
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f= 9.375 GHz
σ_l= 0.0005093
σ= 1.0
L= 10.0
ℓ= 0.0102
ϵ_r= 7.38
G= 0.5
$k\sigma_l$= 0.1
$k\ell$= 2.0
10
0
-10
-20
-30
-40
-50
-60
10
20
30
40
50
60
70
θ(°)


σ^o_{VH}(db)
19
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f = 9.375 GHz
σ_l = 0.0005093
σ = 1.0
L = 10.0
l = 0.0102
ϵ_r = 7.38
G = 0.5
$k\sigma_l$ = 0.1
kl = 2.0
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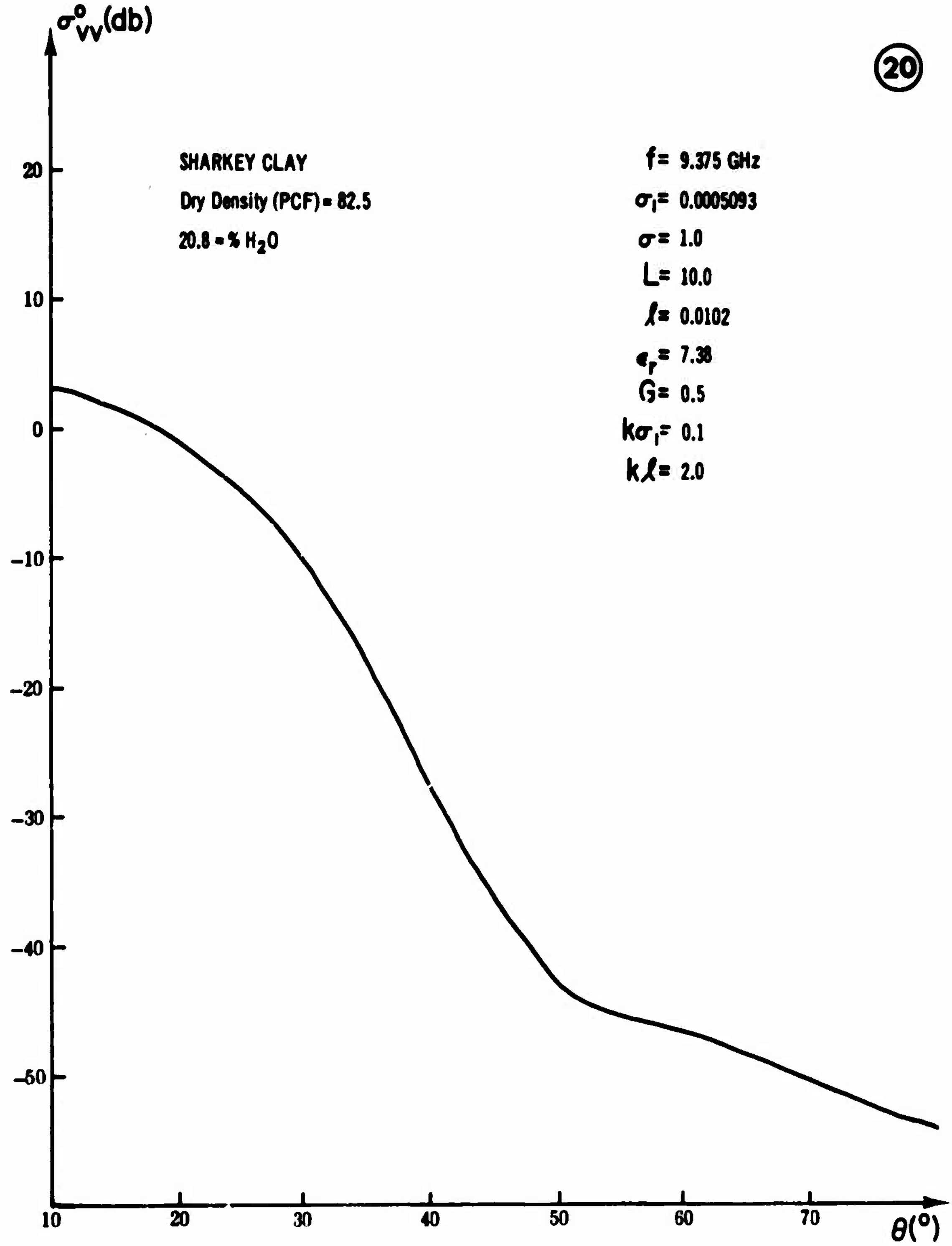
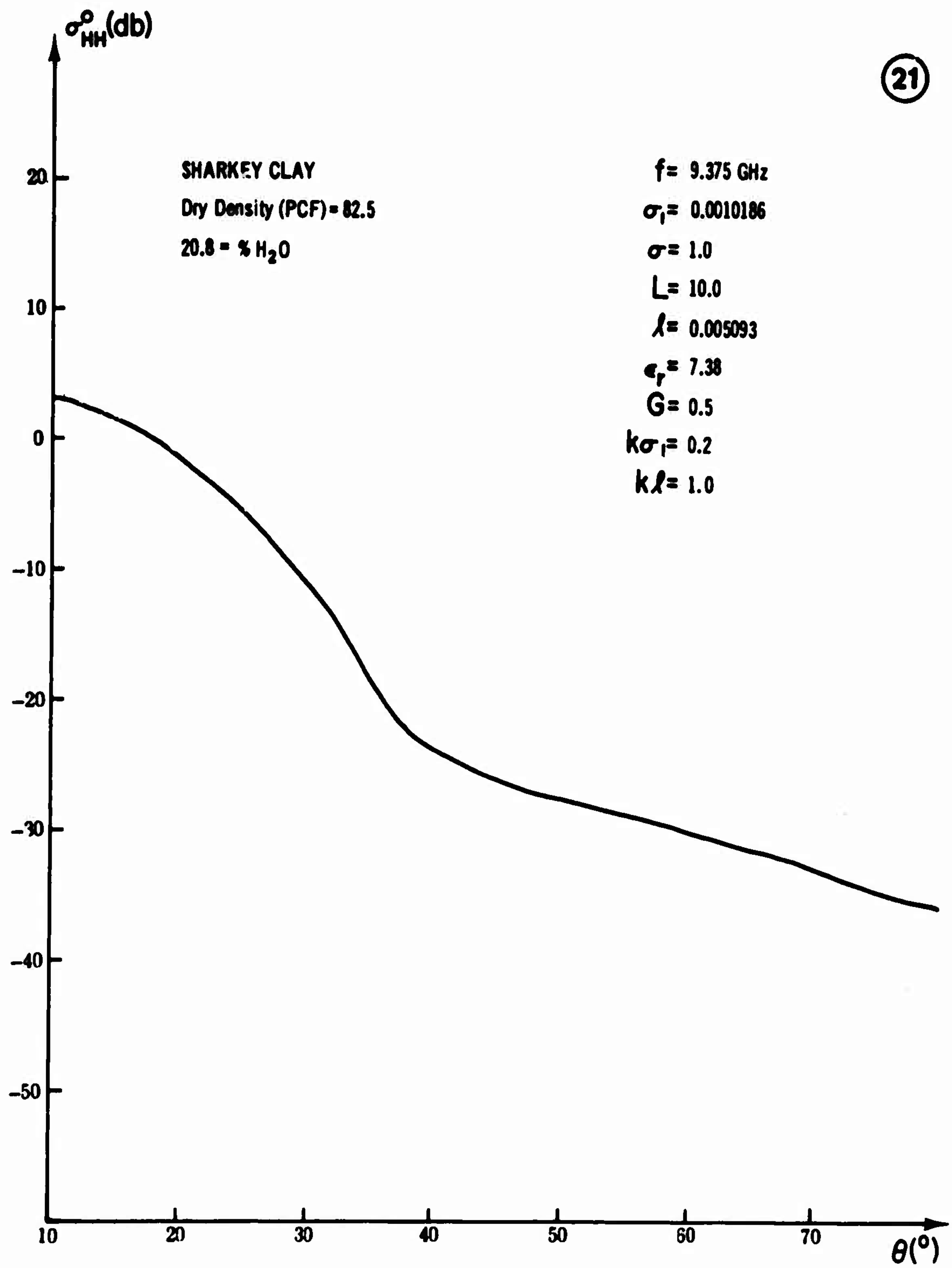
0
-10
-20
-30
-40
-50
-60
10
20
30
40
50
60
70
θ(°)



σ^o_{VV}(db)
20
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f = 9.375 GHz
σ_l = 0.0005093
σ = 1.0
L = 10.0
ℓ = 0.0102
ϵ_r = 7.38
G = 0.5
$k\sigma_l$ = 0.1
$k\ell$ = 2.0
20
10
0
-10
-20
-30
-40
-50
10
20
30
40
50
60
70
θ(°)



σ^o_{HH}(db)
21
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f= 9.375 GHz
σ_l= 0.0010186
σ= 1.0
L= 10.0
ℓ= 0.005093
ϵ_r= 7.38
G= 0.5
$k\sigma_l$= 0.2
$k\ell$= 1.0
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10
0
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-20
-30
-40
-50
10
20
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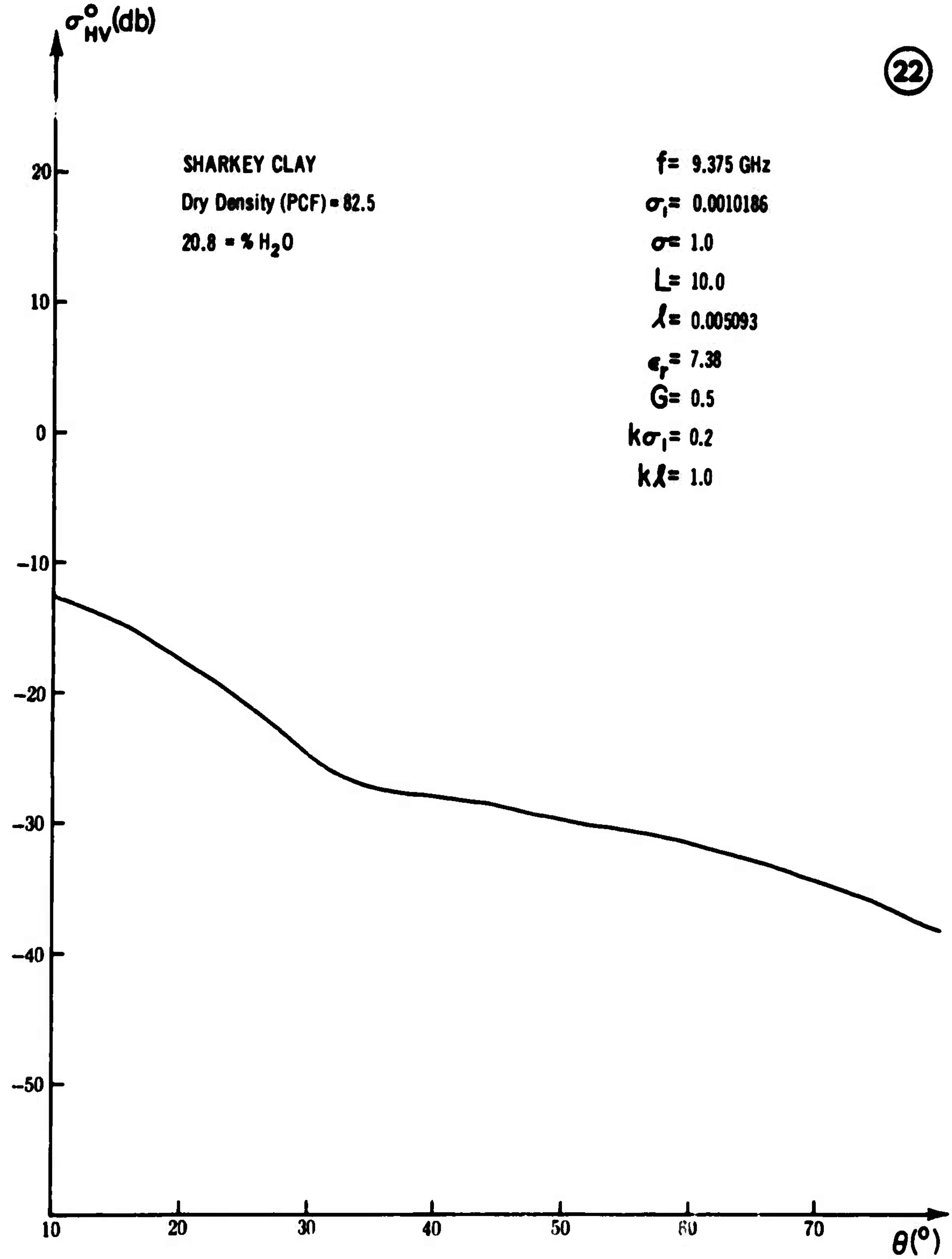
40
50
60
70
θ(°)



σ^{o}_{HV}(db)
22
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f= 9.375 GHz
σ_1= 0.0010186
σ= 1.0
L= 10.0
ℓ= 0.005093
ϵ_r= 7.38
G= 0.5
$k\sigma_1$= 0.2
$k\ell$= 1.0
20
10
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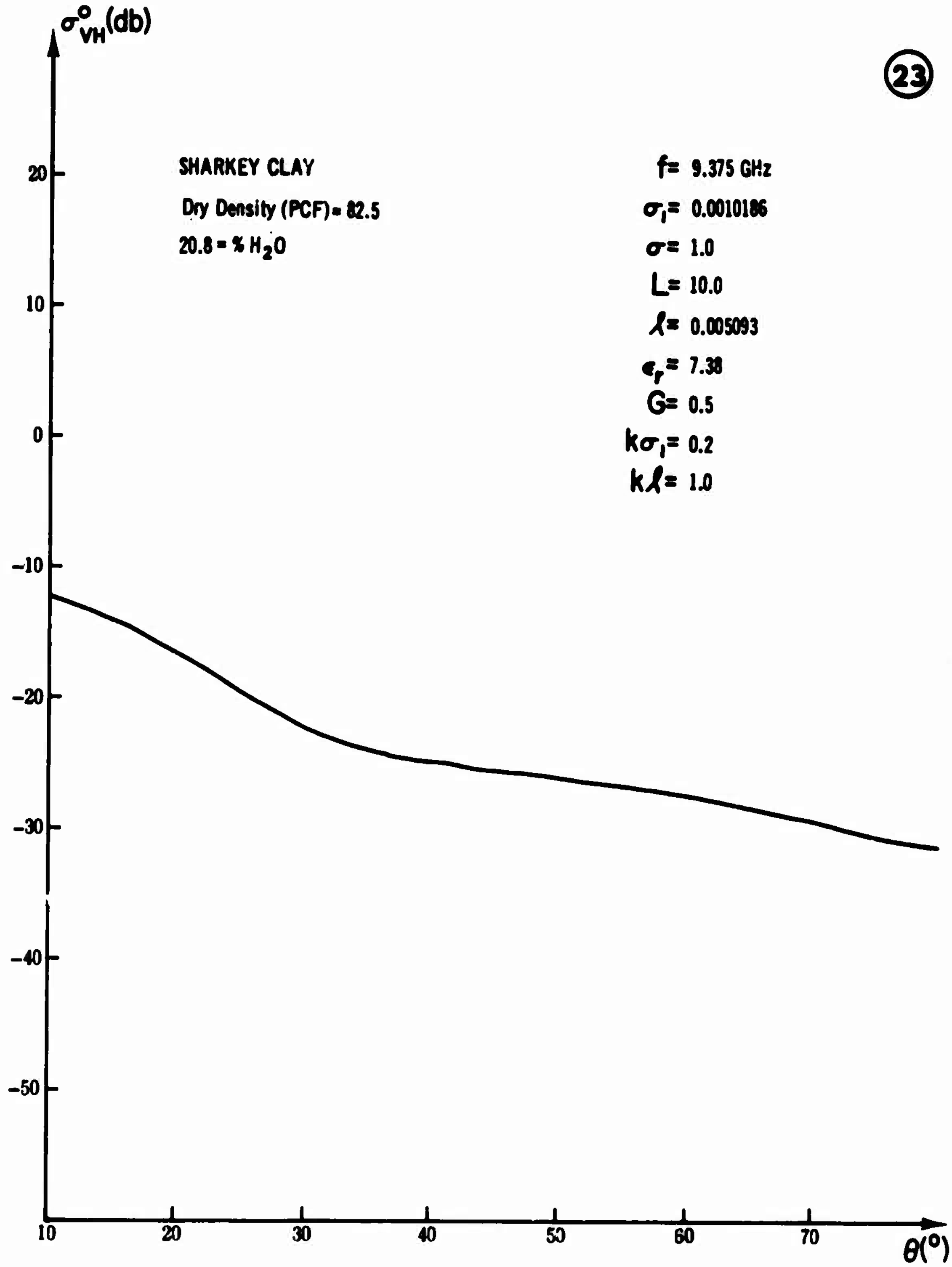
-10
-20
-30
-40
-50
10
20
30
40
50
60
70
θ(°)



σ^o_{VH}(db)
23
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f = 9.375 GHz
σ_l = 0.0010186
σ = 1.0
L = 10.0
ℓ = 0.005093
ϵ_r = 7.38
G = 0.5
$k\sigma_l$ = 0.2
$k\ell$ = 1.0
20
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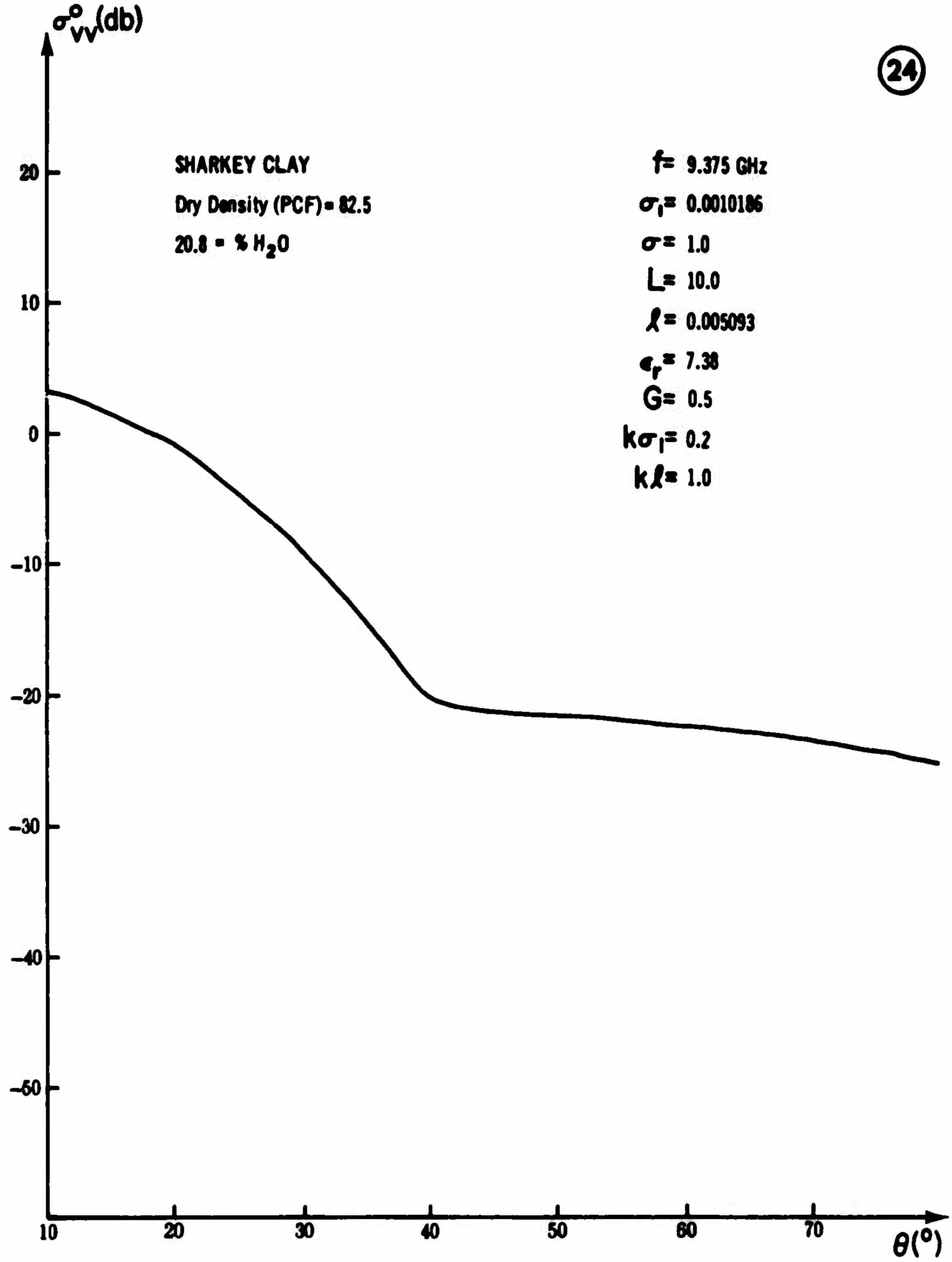
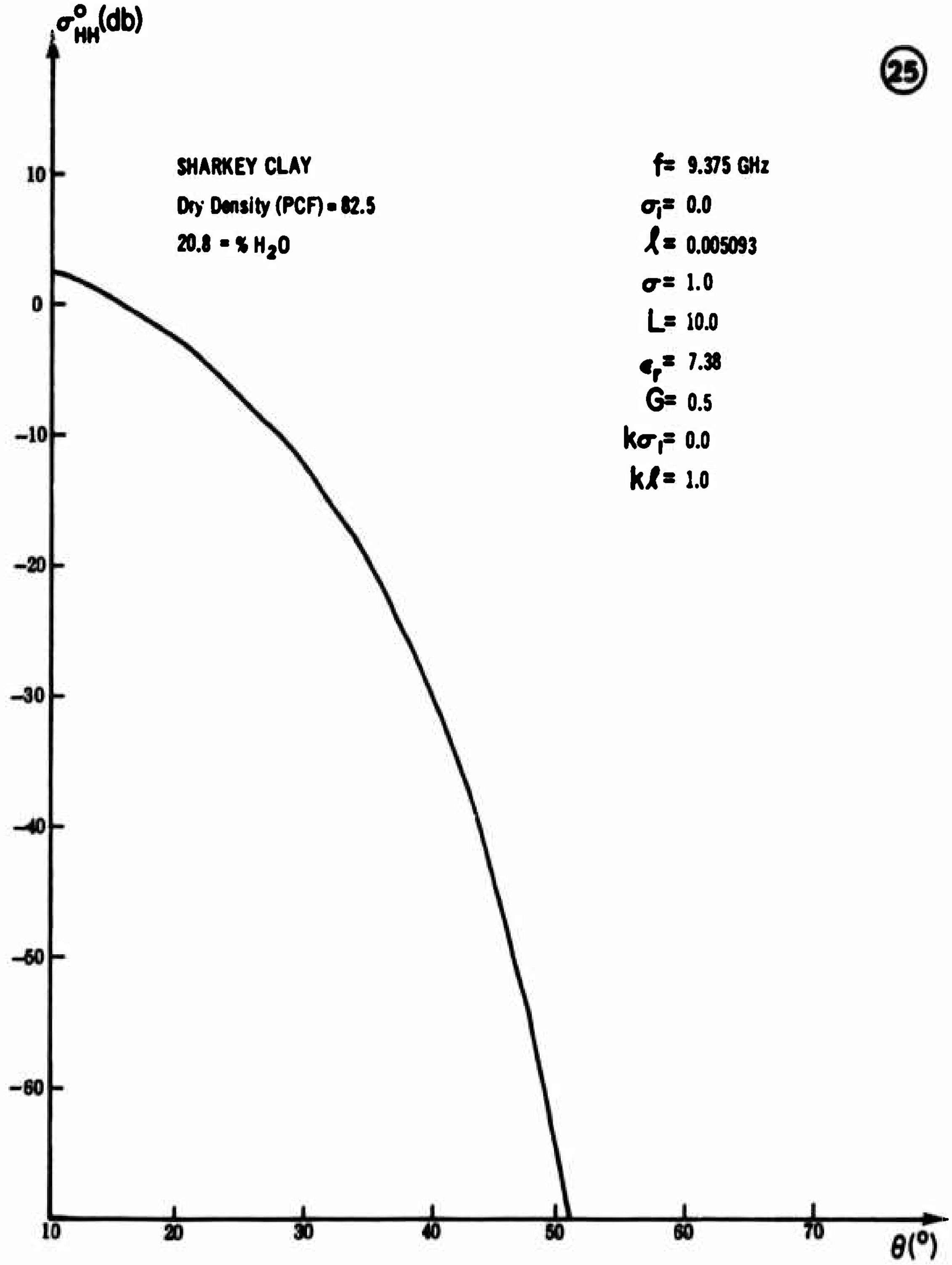
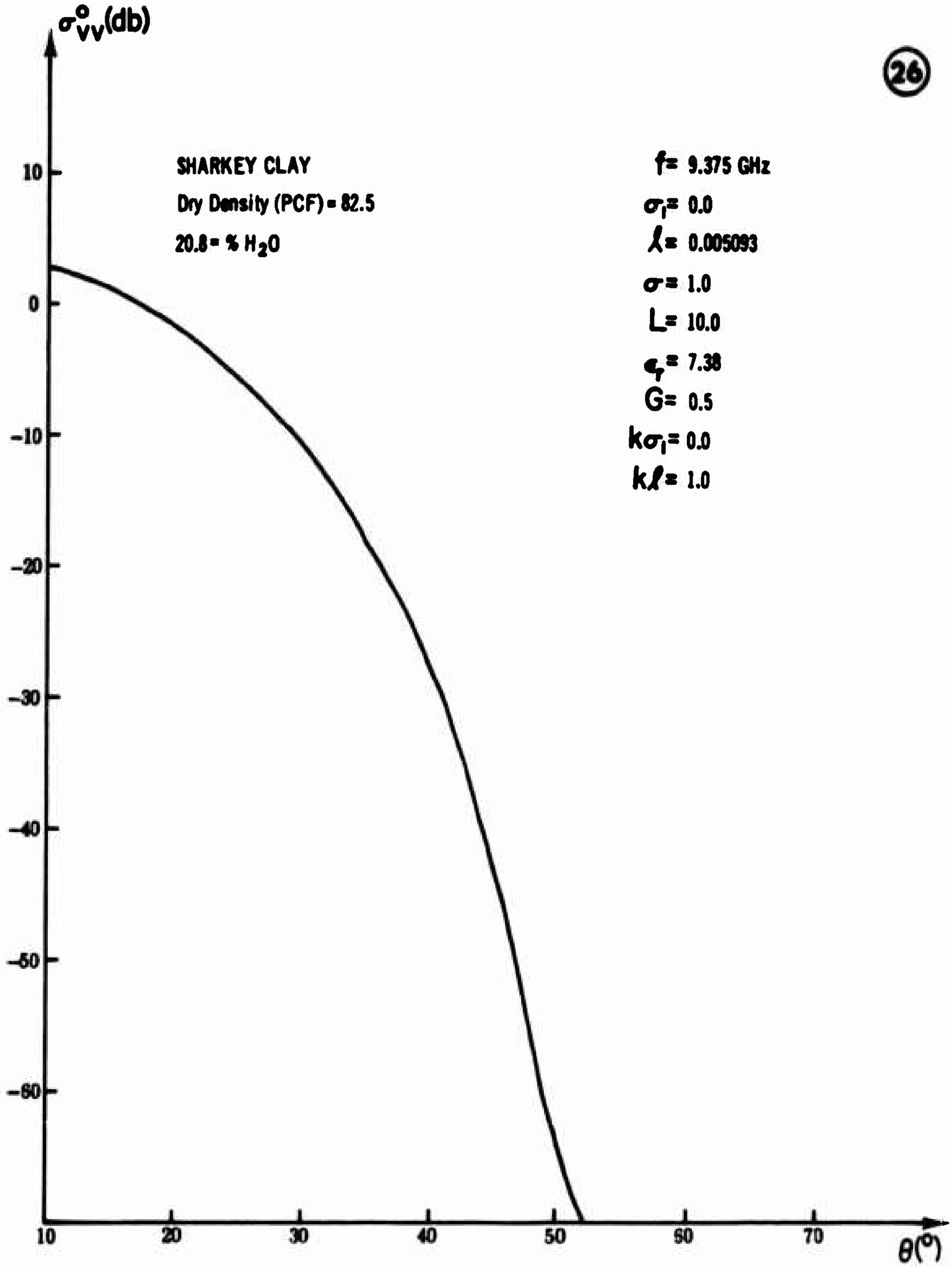
-10
-20
-30
-40
-50
10
20
30
40
50
60
70
θ(°)



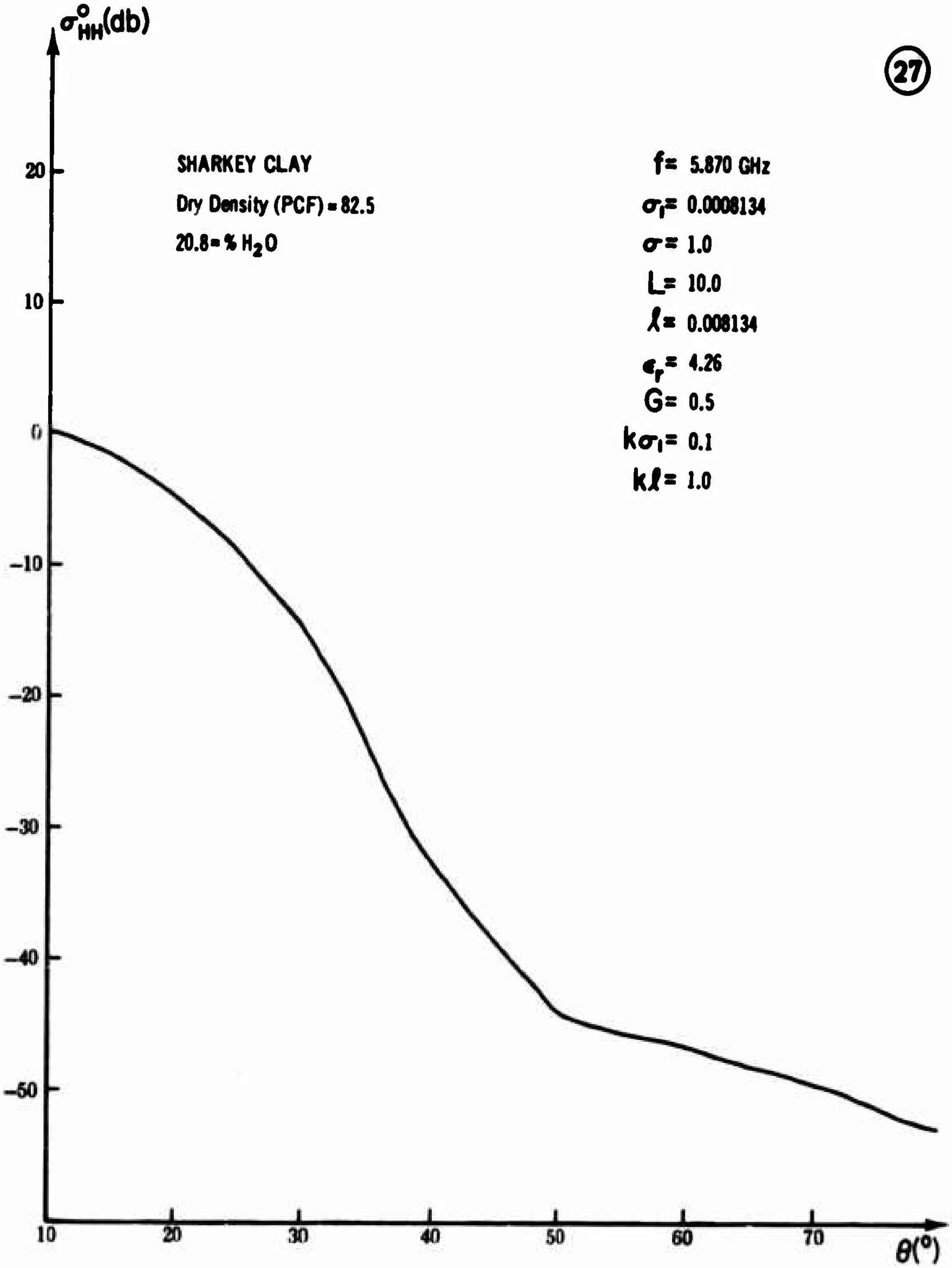
σ^o_{VV}(db)
24
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f = 9.375 GHz
σ_1 = 0.0010186
σ = 1.0
L = 10.0
ℓ = 0.005093
ϵ_r = 7.38
G = 0.5
$k\sigma_1$ = 0.2
$k\ell$ = 1.0
20
10
0
-10
-20
-30
-40
-60
10
20
30
40
50
60
70
θ(°)



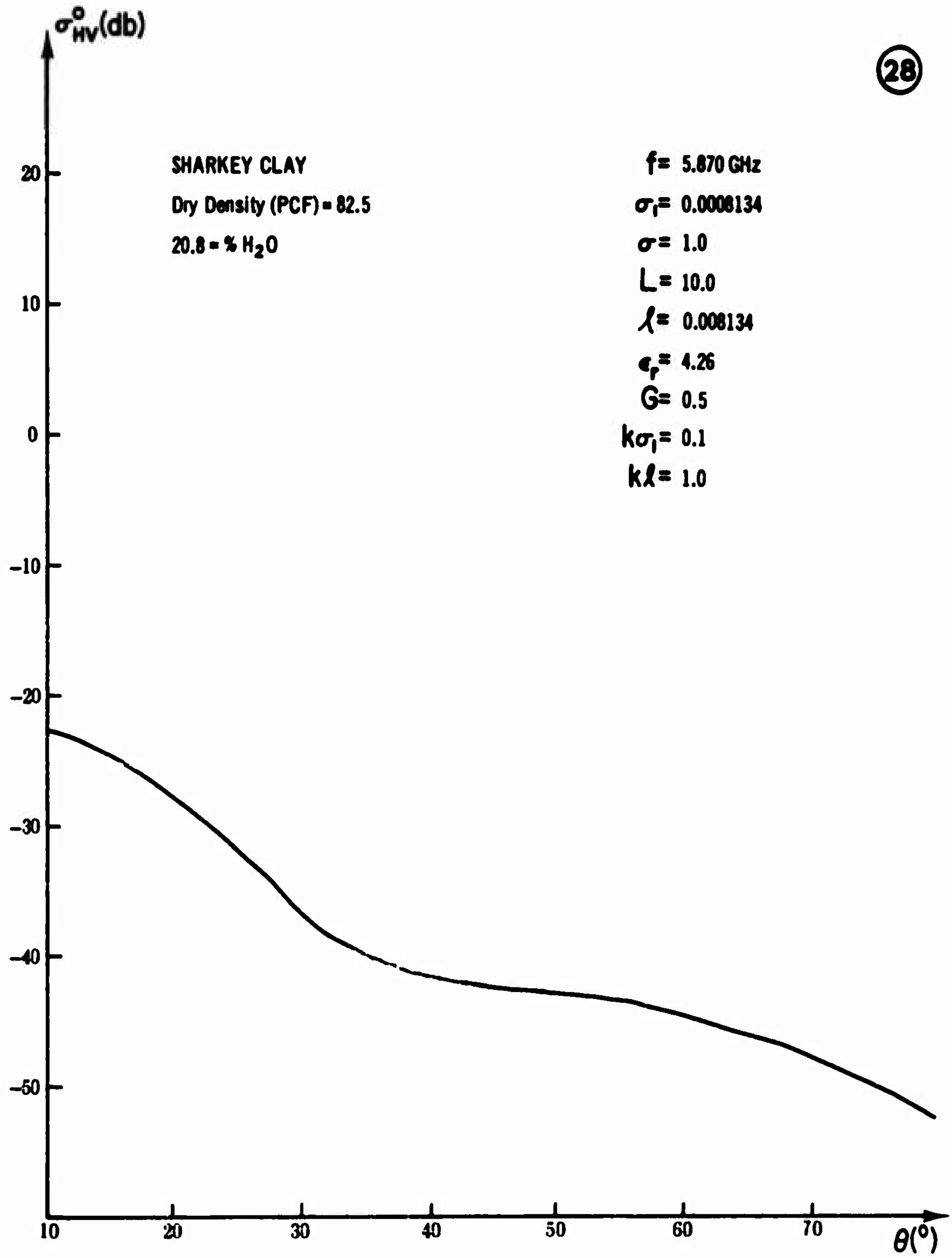
σ^o_{HH}(db)
25
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f= 9.375 GHz
σ_l= 0.0
ℓ= 0.005093
σ= 1.0
L= 10.0
ϵ_r= 7.38
G= 0.5
$k\sigma_l$= 0.0
$k\ell$= 1.0
10
0
-10
-20
-30
-40
-50
-60
10
20
30
40
50
60
70
θ(°)



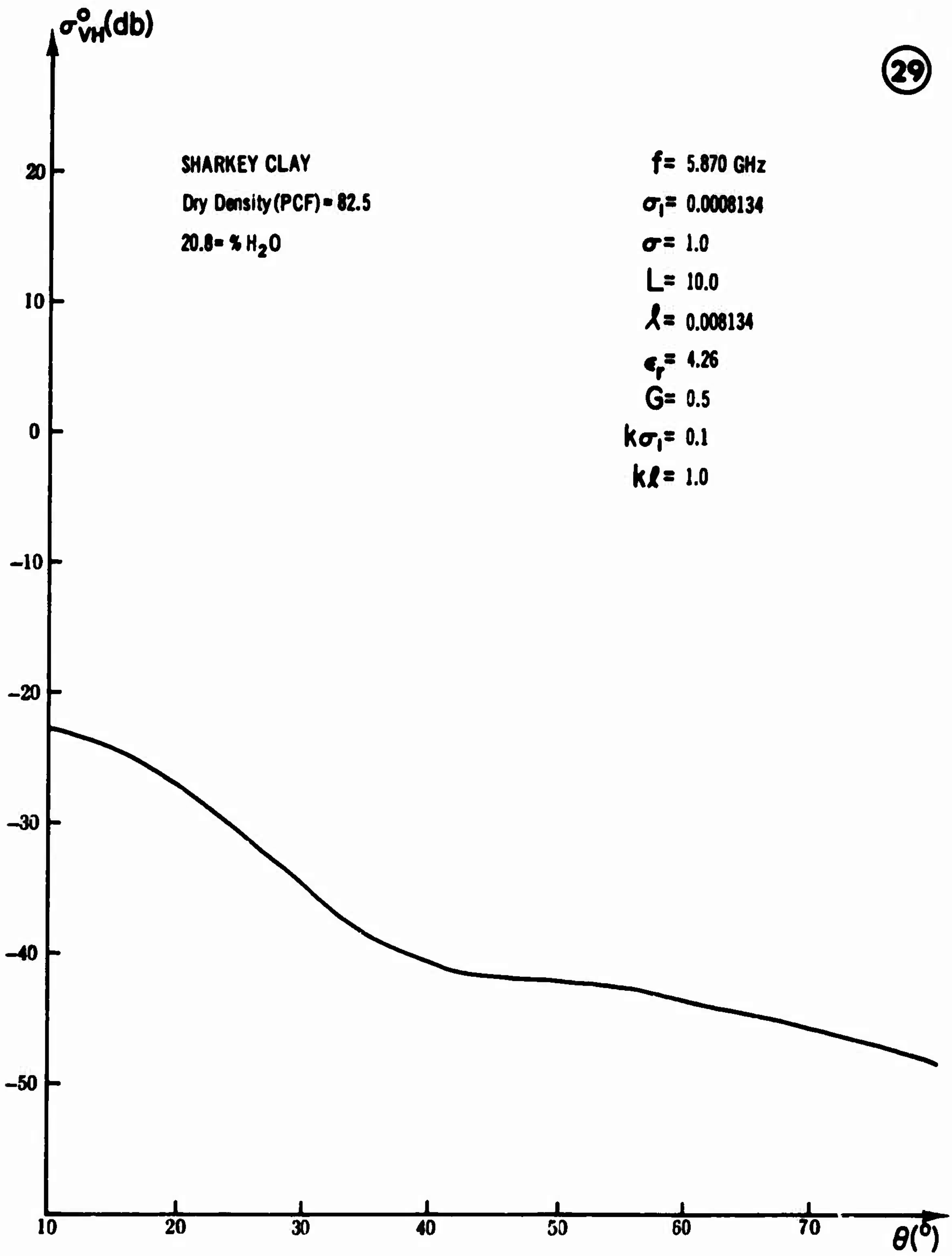
σ^{o}_{VV}(db)
26
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f = 9.375 GHz
σ_l = 0.0
λ = 0.005093
σ = 1.0
L = 10.0
ϵ_r = 7.38
G = 0.5
$k\sigma_l$ = 0.0
$k\ell$ = 1.0
10
0
-10
-20
-30
-40
-50
-60
10
20
30
40
50
60
70
θ(°)



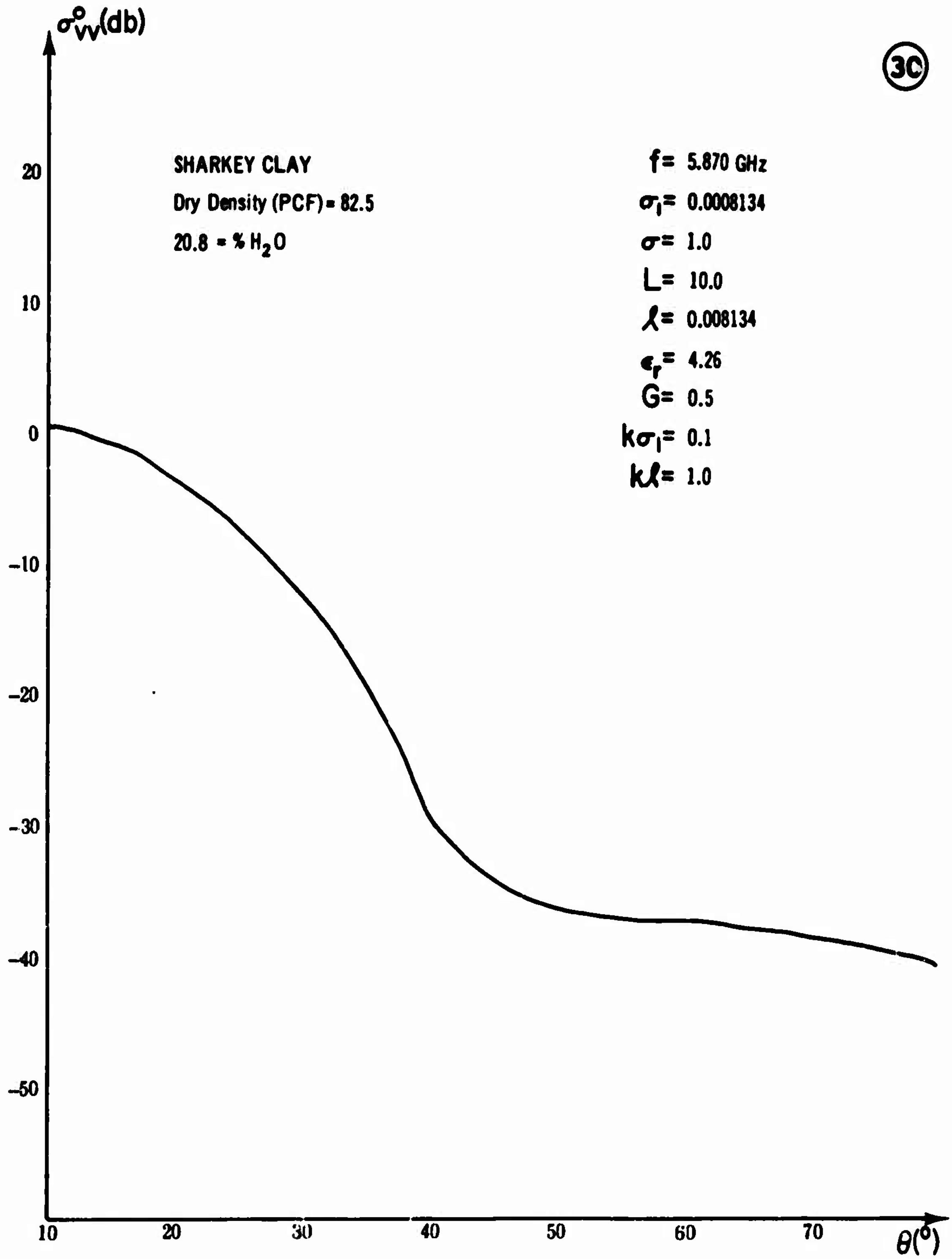
σ^o_{HH}(db)
27
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f = 5.870 GHz
σ_l = 0.0008134
σ = 1.0
L = 10.0
ℓ = 0.008134
ϵ_r = 4.26
G = 0.5
$k\sigma_l$ = 0.1
$k\ell$ = 1.0
20
10
0
−10
−20
−30
−40
−50
10
20
30
40
50
60
70
θ(°)



28
σ^o_{HV}(db)
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f= 5.870 GHz
σ_l= 0.0008134
σ= 1.0
L= 10.0
ℓ= 0.008134
ϵ_r= 4.26
G= 0.5
$k\sigma_l$= 0.1
$k\ell$= 1.0
20
10
0
-10
-20
-30
-40
-50
10
20
30
40
50
60
70
θ(°)



σ^{o}_{VH}(db)
29
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f = 5.870 GHz
σ_1 = 0.0008134
σ = 1.0
L = 10.0
ℓ = 0.008134
ϵ_r = 4.26
G = 0.5
$k\sigma_1$ = 0.1
$k\ell$ = 1.0
20
10
0
-10
-20
-30
-40
-50
10
20
30
40
50
60
70
θ(°)

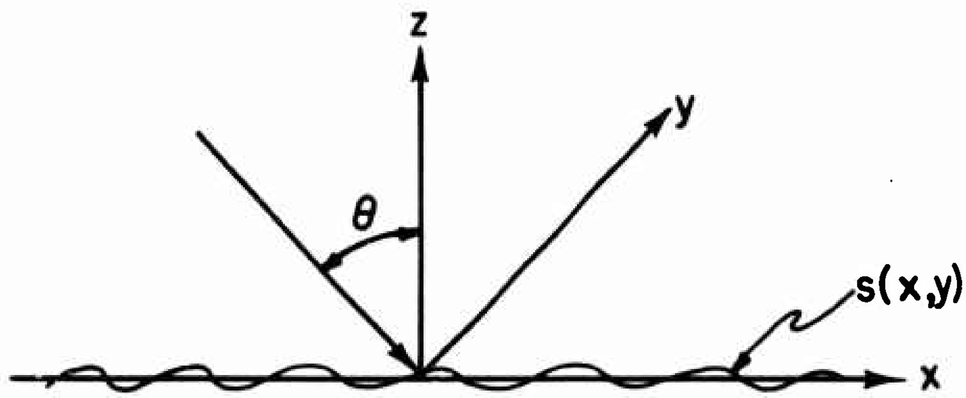


σ^o_{VV}(db)
30
SHARKEY CLAY
Dry Density (PCF) = 82.5
20.8 = % H_2O
f = 5.870 GHz
σ_l = 0.0008134
σ = 1.0
L = 10.0
ℓ = 0.008134
ϵ_r = 4.26
G = 0.5
$k\sigma_l$ = 0.1
$k\ell$ = 1.0
20
10
0
-10
-20
-30
-40
-50
10
20
30
40
50
60
70
$\theta(^o)$

APPENDIX A

DERIVATION OF THE SCATTERED FIELDS FROM A SLIGHTLY ROUGH SURFACE FOR VERTICAL POLARIZATION

Consider a vertically polarized plane wave incident onto a slightly rough surface $s(x,y)$ which is the boundary between two homogeneous dielectric mediums. The geometry of the problem is given below. The surface function $s(x,y)$ represents the z distances from the xy-plane to the surface.



A right-handed rectangular Cartesian coordinate system is set up with the xy-plane forming the average value of $s(x,y)$. The xz-plane coincides with the plane of incidence. The angle θ is the angle of incidence. The conditions that must be made upon $s(x,y)$ are that it be Fourier transformable and that

$$|ks(x,y)| < 1, \text{ where } k = 2\pi/\lambda .$$

The propagation constant in the medium $z < s(x,y)$ is designated k' and is equal to $\omega\sqrt{\mu\epsilon}$. The incident magnetic field is in the $+y$ direction, and the components of the total magnetic field in the space $z > s(x,y)$ can be written as:

$$H_x = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D_x(k_x,k_y)\, EXP_1\, dk_x\, dk_y \tag{1a}$$

$$H_y = \exp(-jkx \sin\theta)\,[\exp(jkz\cos\theta) + R_{\|}\exp(-jkz\cos\theta)]$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_y(k_x, k_y)\, EXP_1\, dk_x\, dk_y \tag{1b}$$

$$H_z = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_z(k_x, k_y)\, EXP_1\, dk_x\, dk_y \tag{1c}$$

where $EXP_1 = \exp(jk_x x + jk_y y - jk_z z)$ and $R_{\|}$ is the Fresnel reflection coefficient for a vertically polarized wave. The parameters k_x and k_y are the Fourier transform variables and k_z is:

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2} \qquad \text{when } k^2 > k_x^2 + k_y^2$$

$$k_z = -j\sqrt{k_x^2 + k_y^2 - k^2} \qquad \text{when } k_x^2 + k_y^2 > k^2$$

The total magnetic field in the medium $z < s(x,y)$ can be written as

$$H_x' = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D_x'(k_x, k_y)\, EXP_2\, dk_x\, dk_y \tag{2a}$$

$$H_y' = T_{\|}\exp(-jk'x\sin\phi)\exp(jk'z\cos\phi) + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_y'(k_x, k_y)\, EXP_2\, dk_x\, dk_y \tag{2b}$$

$$H_z' = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_z'(k_x, k_y)\, EXP_2\, dk_x\, dk_y \tag{2c}$$

where $EXP_2 = \exp(jk_x x + jk_y y + jk_z' z)$ and $T_{\|}$ is equal to $1 + R_{\|}$. The angle ϕ is related to θ through Snell's law. The parameter k_z' is defined as follows:

$$k_z' = \sqrt{k'^2 - k_x^2 - k_y^2} \qquad , \text{when } k'^2 > k_x^2 + k_y^2$$

$$k_z' = -j\sqrt{k_x^2 + k_y^2 - k'^2} \qquad , \text{when } k'^2 < k_x^2 + k_y^2 \quad .$$

If the surface were flat so that $s(x,y) = 0$, then the fields that would be obtained are those of equations (1) and (2) with the doubled-integral terms eliminated. The technique of small perturbation requires that the amplitudes be written in a perturbation series, e.g. $D_x = D_{x1} + D_{x2} + D_{x3} + \dots\dots\dots$ (where the dependence upon k_x and k_y has been omitted). The fields near the interface can be written as follows:

$$H_x = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ D_{x1} + (D_{x2} - jk_z z D_{x1}) + \left(D_{x3} - jk_z z D_{x2} - \frac{k_z^2 z^2 D_{x1}}{2} \right) + \dots\dots \right\}$$

$$\cdot \exp(jk_x x + jk_y y)\, dk_x\, dk_y \tag{3a}$$

$$H'_x = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ D'_{x1} + (D'_{x2} + jk'_z z D'_{x1}) + \left(D'_{x3} + jk'_z z D'_{x2} - \frac{k'^2_z z^2 D'_{x1}}{2} \right) + \dots\dots \right\}$$

$$\cdot \exp(jk_x x + jk_y y)\, dk_x\, dk_y \tag{3b}$$

$$H_y = \exp(-jkx \sin\theta) \left[1 + jk_z \cos\theta - \frac{k^2 z^2 \cos^2\theta}{2} + R_{\|} - jR_{\|} kz \cos\theta - \frac{R_{\|} k^2 z^2 \cos^2\theta}{2} \right] + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ G_{y1} + (G_{y2} - jk_z z G_{y1}) + \left(G_{y3} - jk_z z G_{y2} - \frac{k_z^2 z^2 G_{y1}}{2} \right) + \dots\dots\dots \right\}$$

$$\cdot \exp(jk_x x + jk_y y)\, dk_x\, dk_y \tag{3c}$$

$$H'_y = T_{\|} \exp(-jk'x \sin\phi) \left[1 + jk'z \cos\phi - \frac{k'^2 z^2 \cos^2\phi}{2} \right] + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ G'_{y1} + (G'_{y2} + jk'_z z G'_{y1}) + \left(G'_{y3} + jk'_z z G'_{y2} - \frac{k_z^2 z^2 G'_{y1}}{2} \right) + \dots \right\} \cdot \exp(jk_x x + jk_y y)\, dk_x\, dk_y \tag{3d}$$

$$H_z = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ F_{z1} + (F_{z2} - jk_z z F_{z1}) + \left(F_{z3} - jk_z z F_{z2} - \frac{k_z^2 z^2 F_{z1}}{2} \right) + \dots\dots \right\}$$

$$\exp(jk_x x + jk_y y)\, dk_x\, dk_y \tag{3e}$$

$$H'_z = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ F'_{z1} + (F'_{z2} + jk'_z z F'_{z1}) + \left(F'_{z3} + jk'_z z F'_{z2} - \frac{k'^2_z z^2 F'_{z1}}{2} \right) + \dots\dots \right\}$$

$$\cdot \exp(jk_x x + jk_y y)\, dk_x\, dk_y \tag{3f}$$

where the factors $\exp(-jk_z z)$ and $\exp(jk'_z z)$ have been expanded in a series for the case where z is small. At the surface where $z = s(x,y)$, the boundary conditions $\vec{n} \times (\vec{H} - \vec{H}')$ and $\vec{n} \times (\vec{E} - \vec{E}')$ can be written in component form as

$$(H_x - H'_x) + \left(\frac{\partial s}{\partial x}\right)(H_z - H'_z) = 0 \tag{4a}$$

$$(H_y - H'_y) + \left(\frac{\partial s}{\partial y}\right)(H_z - H'_z) = 0 \ . \tag{4b}$$

$$\frac{\eta}{jk}\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right]_{z=s(x,y)} + \frac{j\eta k}{k'^2}\left[\frac{\partial H'_z}{\partial y} - \frac{\partial H'_y}{\partial z}\right]_{z=s(x,y)} +$$

$$\left(\frac{\partial s}{\partial x}\right)\left[\frac{\eta}{jk}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) + \frac{j\eta k}{k'^2}\left(\frac{\partial H'_y}{\partial x} - \frac{\partial H'_x}{\partial y}\right)\right]_{z=s(x,y)} = 0 \tag{4c}$$

$$\frac{\eta}{jk}\left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right]_{z=s(x,y)} + \frac{j\eta k}{k'^2}\left[\frac{\partial H'_x}{\partial z} - \frac{\partial H'_z}{\partial x}\right]_{z=s(x,y)} + \tag{4d}$$

$$\left(\frac{\partial s}{\partial y}\right)\left[\frac{\eta}{jk}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) + \frac{j\eta k}{k'^2}\left(\frac{\partial H'_y}{\partial x} - \frac{\partial H'_x}{\partial y}\right)\right]_{z=s(x,y)} = 0$$

In order to determine the six unknown amplitudes, six independent equations are needed; so, the four equations above and the two divergence conditions $\vec{\nabla} \cdot \vec{H} = 0$ and $\vec{\nabla} \cdot \vec{H}' = 0$ combine to yield the necessary number of equations. The partial derivatives can be obtained from equations (3). When this is done and only the first order terms are kept, the following results are obtained:

$$\frac{\partial H_x}{\partial x} = j\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_x D_{x1} \exp(jk_x x + jk_y y)\, dk_x\, dk_y \tag{5a}$$

$$\frac{\partial H_y}{\partial y} = j\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_y G_{y1} \exp(jk_x x + jk_y y)\, dk_x\, dk_y \tag{5b}$$

$$\frac{\partial H_z}{\partial z} = -j\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_z F_{z1} \exp(jk_x x + jk_y y)\, dk_x\, dk_y \tag{5c}$$

$$\frac{\partial H'_x}{\partial x} = j \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_x D'_{x1} \exp(jk_x x + jk_y y) dk_x\, dk_y \tag{5d}$$

$$\frac{\partial H'_y}{\partial y} = j \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_y G'_{y1} \exp(jk_x x + jk_y y) dk_x\, dk_y \tag{5e}$$

$$\frac{\partial H'_z}{\partial z} = -j \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k'_z F'_{z1} \exp(jk_x x + jk_y y) dk_x\, dk_y \tag{5f}$$

$$\frac{\partial H_z}{\partial y} = j \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_y F_{z1} \exp(jk_x x + jk_y y) dk_x\, dk_y \tag{5g}$$

$$\frac{\partial H_z}{\partial x} = j \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_x F_{z1} \exp(jk_x x + jk_y y) dk_x\, dk_y \tag{5h}$$

$$\begin{aligned}\frac{\partial H_y}{\partial z} &= \exp(-jkx \sin\theta) \left\{ jk \cos\theta (1 - R_{\|}) - k^2 z \cos^2\theta (1 + R_{\|}) \right\} \\ &\quad - j \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_z G_{y1} \exp(jk_x x + jk_y y) dk_x\, dk_y \end{aligned} \tag{5i}$$

$$\begin{aligned}\frac{\partial H_y}{\partial x} &= -jk \sin\theta \exp(-jkx \sin\theta) \left\{ T_{\|} + jkz \cos\theta (1 - R_{\|}) \right\} \\ &\quad + j \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_x G_{y1} \exp(jk_x x + jk_y y) dk_x\, dk_y \end{aligned} \tag{5j}$$

$$\frac{\partial H_x}{\partial z} = -j \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_z D_{x1} \exp(jk_x x + jk_y y) dk_x\, dk_y \tag{5k}$$

$$\frac{\partial H_z}{\partial x} = j \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_x F_{z1} \exp(jk_x x + jk_y y) dk_x\, dk_y \tag{5l}$$

$$\frac{\partial H'_z}{\partial y} = j\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_y F'_{z1} \exp(jk_x x + jk_y y)\,dk_x\,dk_y \tag{5m}$$

$$\frac{\partial H'_y}{\partial z} = T_{\|} \exp(-jk'x\sin\phi)\left\{jk'\cos\phi - zk'^2\cos^2\phi\right\}$$

$$+\, j\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k'_z G'_{y1} \exp(jk_x x + jk_y y)\,dk_x\,dk_y \tag{5n}$$

$$\frac{\partial H'_x}{\partial z} = j\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k'_z D_{x1} \exp(jk_x x + jk_y y)\,dk_x\,dk_y \tag{5o}$$

$$\frac{\partial H'_z}{\partial x} = j\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_x F'_{z1} \exp(jk_x x + jk_y y)\,dk_x\,dk_y \tag{5p}$$

$$\frac{\partial H'_y}{\partial x} = -jk\sin\theta\, T_{\|} \exp(-jkx\sin\theta)\,[1 + jk'z\cos\phi] \tag{5q}$$

$$+\, j\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_x G'_{y1} \exp(jk_x x + jk_y y)\,dk_x\,dk_y$$

$$\frac{\partial H'_x}{\partial y} = j\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_y D'_{x1} \exp(jk_x x + jk_y y)\,dk_x\,dk_y \tag{5r}$$

Substituting equations (5) into the boundary conditions of equations (4) and the two divergence equations and evaluating the result at $z = s(x,y)$ yields the following six algebraic equations with six unknowns:

$$D_{x1} = D'_{x1} \tag{6a}$$

$$G'_{y1} = G_{y1} + Q_1 \tag{6b}$$

$$k_z F_{z1} = k_y G_{y1} + k_x D_{x1} \tag{6c}$$

$$-k'_z F'_{z1} = k_y G'_{y1} + k_x D'_{x1} \tag{6d}$$

$$Vkk'^2 + \eta k_y k'^2 F_{z1} + \eta k_z k'^2 G_{y1} - \eta k^2 k_y F'_{z1} + \eta k^2 k'_z G'_{y1} + Wkk'^2 = 0 \tag{6e}$$

$$-\eta k'^2 k_z D_{x1} - \eta k'^2 k_x F_{z1} - \eta k^2 k'_z D'_{x1} + \eta k^2 k_x F'_{z1} + kk'^2 S_o = 0 \tag{6f}$$

where:
$$Q_1 = \frac{j(k^2 - k'^2)(1 - R_{\|})\cos\theta\, S(k_x + k\sin\theta, k_y)}{2\pi k}$$

$$V = -jT_{\|}\eta k \sin^2\theta (k^2 - k'^2)\, S(k_x + k\sin\theta, k_y)/2\pi k'^2$$

$$W = j(k_x + k\sin\theta)\,\eta T_{\|} \sin\theta\, (k^2 - k'^2)\, S(k_x + k\sin\theta, k_y)/2\pi k'^2$$

$$S_o = jk_y \eta T_{\|} \sin\theta\, (k^2 - k'^2)\, S(k_x + k\sin\theta, k_y)/2\pi k'^2$$

$$S(k_x + k\sin\theta, k_y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(x,y) \exp\left[-j(k_x + k\sin\theta)x - jk_y y\right] dy\, dx$$

Use has been made of the relation $(k'^2\cos\theta - R_{\|}\, k'^2\cos\theta)/k = T_{\|} k' \cos\phi$. For this analysis, interest is only in the fields above the interface so that solutions need only be found for the terms D_{x1}, G_{y1}, and F_{z1} . Using equations (6), solutions can be obtained for these three terms:

$$D_{x1} = \frac{B_o a_{11} - A_o a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

$$G_{y1} = \frac{A_o a_{22} - B_o a_{12}}{a_{11} a_{22} - a_{12} a_{21}}$$

$$F_{z1} = \frac{k_y G_{y1} + k_x D_{x1}}{k_z}$$

where

$$A_o = -\eta k^2 k_y^2 k_z Q_1 - \eta k_z k^2 k_z'^2 Q_1 - Wk_z k'_z kk'^2 - Vk_z k'_z kk'^2$$

$$B_o = k_z k'_z kk'^2 S_o - \eta k^2 k_x k_y k_z Q_1$$

$$a_{11} = \eta k_y^2 k'_z k'^2 + \eta k_z^2 k'_z k'^2 + \eta k^2 k_y^2 k_z + \eta k^2 k_z'^2 k_z$$

$$a_{12} = \eta k_y k'_z k'^2 k_x + \eta k^2 k_y k_z k_x$$

$$a_{21} = \eta k'^2 k_x k'_z k_y + \eta k^2 k_x k_z k_y$$

$$a_{22} = \eta k'^2 k_z^2 k'_z + \eta k'^2 k_x^2 k'_z + \eta k^2 k_z k_z'^2 + \eta k^2 k_x^2 k_z$$

APPENDIX B

DERIVATION OF AVERAGES USED TO CALCULATE MEAN BACKSCATTERED POWER

This appendix presents the derivation of the necessary averages that were used to determine expressions for mean powers. The two-dimensional gaussian distribution of two random variables z and z' with zero means is

$$p(z,z') = \frac{1}{2\pi\sigma^2\sqrt{1-C^2}} \exp\left[-\frac{z^2 - 2Czz' + z'^2}{2\sigma^2(1-C^2)}\right]$$

C is the autocorrelation coefficient;

σ^2 is the variance.

The two-dimensional characteristic function $\chi(v_1, v_2)$ of the distribution $p(z,z')$ is

$$\chi(v_1, v_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{jv_1 z} e^{jv_2 z'} p(z,z')\, dz\, dz'$$

$$\chi(v_1, v_2) = \exp\left\{-\frac{1}{2}\sigma^2(v_1^2 + 2Cv_1v_2 + v_2^2)\right\}$$

The first function that will be averaged is

$$e^{jk_1(z-z')}$$

Where k_1 is an arbitrary function of the angle of incidence θ

$$\langle e^{jk_1(z-z')}\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{jk_1 z} e^{-jk_1 z'} p(z,z')\, dz\, dz'$$

By analogy with the characteristic function $\chi(v_1, v_2)$, it can be seen that $v_1 \to k_1$ and $v_2 \to -k_1$. The required average is then

$$\langle e^{jk_1(z-z')}\rangle = \exp[-k_1^2\sigma^2(1-C)]\ .$$

In order to calculate $\langle Z_x e^{jk_1(z-z')}\rangle$, it is necessary to rewrite the quantity inside the brackets as

$$\frac{1}{jk_1}\frac{\partial}{\partial x}\exp[jk_1(Z-Z')]$$

$$<Z_x e^{jk_1(Z-Z')}> = \frac{1}{jk_1}\frac{\partial}{\partial x} <\exp\left\{jk_1(Z-Z')\right\}>$$
$$= \frac{1}{jk_1}\frac{\partial}{\partial u}\left\{\exp[-k_1^2\sigma^2(1-C)]\right\}\frac{\partial u}{\partial x}$$

where $u = x - x'$ and $v = y - y'$

$$<Z_x e^{jk_1(Z-Z')}> = -jk_1\sigma^2\frac{\partial C}{\partial u}\exp[-k_1^2\sigma^2(1-C)] \, .$$

The average $<Z_y e^{jk_1(Z-Z')}>$ can be computed in exactly the same manner as above, but the work is done with y and v instead of x and u. When this is done, the result becomes

$$<Z_y e^{jk_1(z-z')}> = -jk_1\sigma^2\frac{\partial C}{\partial v}\exp[-k_1^2\sigma^2(1-C)] \, .$$

To calculate the average $<Z_x Z'_x e^{jk_1(z-z')}>$, it is necessary to rewrite the quantity in the brackets as

$$\frac{-1}{jk_1}\frac{\partial}{\partial x'}\frac{1}{jk_1}\frac{\partial}{\partial x}\left[e^{jk_1(Z-Z')}\right]$$

$$<Z_x Z'_x e^{jk_1(z-z')}> = \frac{1}{k_1^2}\frac{\partial^2}{\partial u^2}<e^{jk_1(Z-Z')}>\left(\frac{\partial u}{\partial x'}\right)\left(\frac{\partial u}{\partial x}\right)$$

$$= -\frac{1}{k_1^2}\frac{\partial}{\partial u}\left\{k_1^2\sigma^2\frac{\partial C}{\partial u}\exp[-k_1^2\sigma^2(1-C)]\right\}$$

$$= -\sigma^2\left[\frac{\partial^2 C}{\partial u^2} + k_1^2\sigma^2\left(\frac{\partial C}{\partial u}\right)^2\right]\exp\left\{-k_1^2\sigma^2(1-C)\right\}$$

The rest of the averages which depend solely on the large surface undulations can be computed in a manner similar to the above methods and the results will be stated here

$$<Z_x Z'_y e^{jk_1(Z-Z')}> = <Z'_x Z_y e^{jk_1(Z-Z')}> =$$
$$-\sigma^2\left\{\frac{\partial^2 C}{\partial u\partial v} + k_1^2\sigma^2\left(\frac{\partial C}{\partial u}\right)\left(\frac{\partial C}{\partial v}\right)\right\}\exp[-k_1^2\sigma^2(1-C)]$$

$$<Z'_x e^{jk_1(Z-Z')}> = <Z_x e^{jk_1(Z-Z')}> = -jk_1\sigma^2 \frac{\partial C}{\partial u} \exp[-k_1^2\sigma^2(1-C)]$$

$$<Z'_y e^{jk_1(Z-Z')}> = <Z_y e^{jk_1(Z-Z')}> = -jk_1\sigma^2 \frac{\partial C}{\partial v} \exp[-k_1^2\sigma^2(1-C)]$$

$$<Z_y Z'_y e^{jk_1(Z-Z')}> = -\sigma^2 \left\{ \frac{\partial^2 C}{\partial v^2} + k_1^2\sigma^2\left(\frac{\partial C}{\partial v}\right)^2 \right\} \exp[-k_1^2\sigma^2(1-C)]$$

The averages that occur with the slightly rough surface $s(x,y)$ are much more complicated, however, if we could compute $<SS^*\exp(jas - ja^*s')>$, all the other averages can be determined by methods used for the large undulations alone. The Fourier transform of $s(x,y)$ is $S(k_x, k_y)$ and can be written as

$$S(k_x, k_y) = \frac{1}{2\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(x,y)\, e^{-jk_x x}\, e^{-jk_y y}\, dy\, dx$$

$$<SS^*\exp(jas - ja^*s')> = <\frac{1}{4\pi^2} \iiiint ss' e^{-jk_x x}\, e^{jk'_x x'}\, e^{-jk_y y}\, e^{jk'_y y'}\, dy\, dx\, dy'\, dx'$$

$$\cdot \exp(jas - ja^*s')>$$

$$= <\frac{1}{4\pi^2} \iiiint s_1 s_2\, e^{-jk_x x}\, e^{jk'_x x'}\, e^{-jk_y y}\, e^{jk'_y y'} \exp(jas_3 - ja^*s_4)\, dy\, dx\, dy'\, dx'>$$

$$s_1 = s(x,y) \quad s_2 = s(x',y') \quad s_3 = s(x'', y'') \quad s_4 = s(x''', y''')$$

$$= \frac{1}{4\pi^2} \iiiint e^{-jk_x x}\, e^{jk'_x x'}\, e^{-jk_y y}\, e^{jk'_y y'} <s_1 s_2 \exp(jas_3 - ja^*s_4)>\, dy\, dx\, dy'\, dx'$$

The multivariate gaussian characteristic function is given by the following equation:

$$M(jv_1, jv_2, jv_3, jv_4) = \iiiint e^{jv_1 s_1 + jv_2 s_2 + jv_3 s_3 + jv_4 s_4}\, p(s_1, s_2, s_3, s_4)\, ds_1\, ds_2\, ds_3\, ds_4$$

the quantity $p(s_1, s_2, s_3, s_4)$ is the multivariate gaussian distribution function. From Davenport and Root,[13] the multivariate characteristic function can also be written as the following equation when the means of all four variables are zero:

$$M(jv_1, jv_2, jv_3, jv_4) = \exp\left\{ -\frac{1}{2} \sum_{m=1}^{4} \sum_{n=1}^{4} \lambda_{mn} v_n v_m \right\} . \qquad \lambda_{mn} = \sigma_n \sigma_m C_{mn}$$

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[13]Davenport and Root, *An Introduction to the Theory of Random Signals and Noise*, McGraw-Hill, 1958, p. 153.

$$\left.\frac{-\partial^2 M}{\partial v_1 \partial v_2}\right|_{v_1 = v_2 = 0} = \iiiint s_1 s_2 e^{jv_3 s_3} e^{jv_4 s_4} p(s_1, s_2, s_3, s_4)\, ds_1\, ds_2\, ds_3\, ds_4$$

$$= \langle s_1 s_2 \exp(jv_3 s_3 + jv_4 s_4) \rangle$$

$$M(jv_1, jv_2, jv_3, jv_4) = \exp\left\{-\frac{1}{2}[\lambda_{11} v_1^2 + \lambda_{12} v_2 v_1 + \lambda_{13} v_3 v_1 + \lambda_{14} v_1 v_4\right.$$

$$+ \lambda_{21} v_1 v_2 + \lambda_{22} v_2^2 + \lambda_{23} v_2 v_3 + \lambda_{24} v_2 v_4 + \lambda_{31} v_1 v_3 + \lambda_{32} v_3 v_2$$

$$\left. + \lambda_{33} v_3^2 + \lambda_{34} v_3 v_4 + \lambda_{41} v_1 v_4 + \lambda_{42} v_4 v_2 + \lambda_{43} v_4 v_3 + \lambda_{44} v_4^2]\right\}$$

When $m = n$, $C_{mn} = 1$. For a stationary process $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2$. Then $\lambda_{11} = \lambda_{22} = \lambda_{33} = \lambda_{44}$.

$$M(jv_1, jv_2, jv_3, jv_4) = \exp\left\{\frac{-1}{2}[\lambda_{11} (v_1^2 + v_2^2 + v_3^2 + v_4^2) + 2\lambda_{12} v_1 v_2\right.$$

$$\left. + 2\lambda_{13} v_1 v_3 + 2\lambda_{14} v_1 v_4 + 2\lambda_{23} v_2 v_3 + 2\lambda_{24} v_2 v_4 + 2\lambda_{34} v_3 v_4]\right\}$$

$$\frac{\partial^2 M}{\partial v_2 \partial v_1} = -\lambda_{12} \exp\left\{-\frac{1}{2}[\lambda_{11}(v_1^2 + v_2^2 + v_3^2 + v_4^2) + 2\lambda_{12} v_1 v_2 +\right.$$

$$\left. 2\lambda_{13} v_1 v_3 + 2\lambda_{14} v_1 v_4 + 2\lambda_{23} v_2 v_3 + 2\lambda_{24} v_2 v_4 + 2\lambda_{34} v_3 v_4]\right\}$$

$$+ [v_1 \lambda_{11} + \lambda_{12} v_2 + \lambda_{13} v_3 + \lambda_{14} v_4][\lambda_{11} v_2 + \lambda_{12} v_1 + \lambda_{23} v_3 + \lambda_{24} v_4]$$

$$\cdot \exp\left\{-\frac{1}{2}[\lambda_{11}(v_1^2 + v_2^2 + v_3^2 + v_4^2) + 2\lambda_{12} v_1 v_2 + 2\lambda_{13} v_1 v_3 +\right.$$

$$\left. 2\lambda_{14} v_1 v_4 + 2\lambda_{23} v_2 v_3 + 2\lambda_{24} v_2 v_4 + 2\lambda_{34} v_3 v_4]\right\}$$

$$\left.\frac{-\partial^2 M}{\partial v_2 \partial v_1}\right|_{v_1 = v_2 = 0} = \left\{+\lambda_{12} - (\lambda_{13} v_3 + \lambda_{14} v_4)(\lambda_{23} v_3 + \lambda_{24} v_4)\right\}$$

$$\cdot \exp\left\{-\frac{1}{2}[\lambda_{11}(v_3^2 + v_4^2) + 2\lambda_{34} v_3 v_4]\right\}$$

$$v_3 = a \qquad v_4 = -a^*$$

$$\lambda_{12} = \sigma_1^2 C_{12}, \qquad \lambda_{13} = \sigma_1^2 C_{13}, \qquad \lambda_{14} = \sigma_1^2 C_{14}$$

$$\lambda_{23} = \sigma_1^2 C_{23}, \qquad \lambda_{24} = \sigma_1^2 C_{24}, \qquad \lambda_{34} = \sigma_1^2 C_{34}$$

It can easily be seen that the term $(\lambda_{13}v_3 + \lambda_{14}v_4)(\lambda_{23}v_3 + \lambda_{24}v_4)$ is of the order of σ_1^4, while the term λ_{12} is of the order of σ_1^2. The variance of the slightly rough surface (σ_1^2) will always be smaller than one for the wavelengths to be considered in this report. Therefore, the term $(\lambda_{13}v_3 + \lambda_{14}v_4)(\lambda_{23}v_3 + \lambda_{24}v_4)$ can be neglected with respect to λ_{12}. The above equation can then be written as

$$\left.\frac{-\partial^2 M}{\partial v_2 \partial v_1}\right|_{v_1 = v_2 = 0} = \sigma_1^2 C_{12} \exp\left\{-\frac{1}{2}\left[\sigma_1^2 (a^2 + a^{*2}) - 2aa^* C_{34}\sigma_1^2\right]\right\}$$

$$<SS^* \exp(jas - ja^*s')> \approx \frac{1}{4\pi^2}\iiiint e^{-jk_x x} e^{-jk_y y} e^{jk'_x x'} e^{jk'_y y'} \sigma_1^2 C_{12}(x - x', y - y')$$

$$\cdot\ dy\,dx\,dy'dx' \exp\left\{-\frac{1}{2}\sigma_1^2\left[a^2 + a^{*2} - 2aa^* C_{34}\right]\right\}$$

$$x - x' = u \qquad y - y' = v \qquad C_{12}(u, v) = C_1(u,v)$$

$$<SS^* \exp(jas - ja^*s)> \approx \frac{1}{4\pi^2}\iiiint e^{-jk_x u} e^{-jk_y v} e^{jx'(k'_x - k_x)} e^{jy'(k'_y - k_y)} \sigma_1^2$$

$$\cdot\ C_1(u,v)\,du\,dv\,dx'dy' \cdot \exp\left\{-\frac{1}{2}\sigma_1^2\left[a^2 + a^{*2} - 2aa^* C_{34}\right]\right\}$$

since the surface is the same in all cases with respect to correlation $C_{34} = C_1(u,v)$. Integrating the above equation in x' and y', the Dirac delta functions appear.

$$<SS^* \exp(jas - ja^*s)> \approx$$

$$\delta(k'_x - k_x)\,\delta(k'_y - k_y)\,\sigma_1^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_1(u,v)\, e^{-jk_x u} e^{-jk_y v}\, du\,dv$$

$$\cdot\ \exp\left\{-\frac{1}{2}\sigma_1^2\left[a^2 + a^{*2} - 2aa^* C_1\right]\right\}$$

The double integral in the last equation represents the roughness spectrum $W(k_x, k_y)$:

$$W(k_x, k_y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_1(u,v)\, e^{-jk_x u} e^{-jk_y v}\, du\,dv$$

The final average is then

$$<SS^* \exp(jas - ja^*s)> \approx 2\pi\delta(k'_x - k_x)\,\delta(k'_y - k_y)\,\sigma_1^2\, W(k_x, k_y)$$

$$\cdot\ \exp\left\{-\frac{1}{2}\sigma_1^2\left[a^2 + a^{*2} - 2aa^* C_1\right]\right\}$$

When $C_1(u,v)$ is gaussian and of the form

$$C_1(u,v) = e^{-(u^2+v^2)/l^2}$$

then $W(k_x,k_y)$ is

$$W(k_x,k_y) = \frac{l^2}{2}\exp\left\{-[k_x^2+k_y^2]l^2/4\right\}$$

In the derivations of the analysis section of the report, k_x must be replaced by $k_x + k\sin\theta'$ and is actually approximated with $k_x + k\sin\theta$.

The calculation of the average $\langle s_x SS^* \exp(jas - ja^*s')\rangle$ can be performed easily now as shown below:

$$\langle s_x SS^*\exp(jas - ja^*s')\rangle \approx \frac{1}{ja}\frac{\partial}{\partial x}\langle SS^*\exp(jas - ja^*s')\rangle$$

$$\approx \frac{1}{ja}\frac{\partial}{\partial u}\langle SS^*\exp(jas - ja^*s')\rangle\frac{\partial u}{\partial x}$$

$$\langle s_x SS^*\exp(jas - ja^*s')\rangle \approx -\frac{j}{a}\frac{\partial}{\partial u}\left\{2\pi\delta(k_y' - k_y)\delta(k_x' - k_x)\right.$$

$$\left.\cdot W(k_x,k_y)\sigma_1^2\exp\left\{-\frac{1}{2}\sigma_1^2[a^2 + a^{*2} - 2aa^*C_1]\right\}\right.$$

$$\approx -ja^*\sigma_1^4\frac{\partial C_1}{\partial u}\,2\pi\delta(k_x' - k_x)\delta(k_y' - k_y)W(k_x,k_y)$$

$$\cdot\exp\left\{-\frac{1}{2}\sigma_1^2[a^2 + a^{*2} - 2aa^*C_1]\right\}$$

Using this same technique, the following averages can be computed easily:

$$\langle s_y SS^*\exp(jas - ja^*s')\rangle \approx -j2\pi a^*\sigma_1^4\frac{\partial C_1}{\partial v}\delta(k_x' - k_x)\delta(k_y' - k_y)$$

$$\cdot W(k_x,k_y)\exp\left\{-\frac{1}{2}\sigma_1^2[a^2 + a^{*2} - 2aa^*C_1]\right\}$$

$$\langle s_x' SS^*\exp(jas - ja^*s')\rangle \approx -j2\pi a\,\sigma_1^4\frac{\partial C_1}{\partial u}\delta(k_x' - k_x)\delta(k_y' - k_y)$$

$$\cdot W(k_x,k_y)\exp\left\{-\frac{1}{2}\sigma_1^2[a^2 + a^{*2} - 2aa^*C_1]\right\}$$

$$\langle s'_y SS^* \exp(jas - ja^*s')\rangle \approx -j2\pi a\sigma_1^4 \frac{\partial C_1}{\partial v}\,\delta(k'_x - k_x)\,\delta(k'_y - k_y)$$

$$\cdot\; W(k_x, k_y)\exp\left\{-\frac{1}{2}\sigma_1^2[a^2 + a^{*2} - 2aa^*C_1]\right\}$$

$$\langle s_x s'_x SS^* \exp(jas - ja^*s)\rangle \approx -2\pi\sigma_1^4\,\delta(k'_x - k_x)\,\delta(k'_y - k_y)$$

$$\cdot\; W(k_x, k_y)\cdot\left\{\frac{\partial^2 C_1}{\partial u^2} + \sigma_1^2 aa^*\left(\frac{\partial C_1}{\partial u}\right)^2\right\}$$

$$\cdot\; \exp\left\{-\frac{1}{2}\sigma_1^2[a^2 + a^{*2} - 2aa^*C_1]\right\}$$

$$\langle s_y s'_y SS^* \exp(jas - ja^*s)\rangle \approx -2\pi\sigma_1^4\,\delta(k'_x - k_x)\,\delta(k'_y - k_y)\,W(k_x, k_y)$$

$$\cdot\left\{\frac{\partial^2 C_1}{\partial v^2} + \sigma_1^2 aa^*\left(\frac{\partial C_1}{\partial v}\right)^2\right\}\exp\left\{-\frac{1}{2}\sigma_1^2[a^2 + a^{*2}\right.$$

$$\left. - 2aa^*C_1]\right\}$$

$$\langle s_x s'_y SS^* \exp(jas - ja^*s)\rangle = \langle s_y s'_x SS^* \exp(jas - ja^*s)\rangle \approx -2\pi\sigma_1^4$$

$$\cdot\; \delta(k'_x - k_x)\,\delta(k'_y - k_y)\,W(k_x, k_y)\left\{\frac{\partial^2 C_1}{\partial u\partial v} + \sigma_1^2 aa^*\left(\frac{\partial C_1}{\partial u}\right)\left(\frac{\partial C_1}{\partial v}\right)\right\}$$

$$\cdot\; \exp\left\{-\frac{1}{2}\sigma_1^2[a^2 + a^{*2} - 2aa^*C_1]\right\}$$

All the required averages needed to derive average power have now been determined.

APPENDIX C

EVALUATION OF INTEGRALS

This appendix shows the derivation of some of the integrals needed in the analysis section of the report. The two basic parameters which are used in the needed integrals are α and r. There is only one integral in r that needs to be specified and this can be obtained from Gradshteyn and Ryzhik.[14]

$$\int_0^\infty r^{\nu+1} e^{-\alpha r^2} J_\nu(\beta r)\, dr = \frac{\beta^\nu \exp(-\beta^2/4\alpha)}{(2\alpha)^{\nu+1}}$$

One of the integrals in α which must be calculated is:

$$\int_0^{2\pi} \cos(\alpha)\, e^{-j\beta \sin(\alpha)} d\alpha$$

Let $u = j\beta \sin\alpha$

$du = j\beta \cos\alpha\, d\alpha$

$$\int_0^{2\pi} \cos(\alpha)\, e^{-j\beta \sin\alpha}\, d\alpha = \int \cos\alpha\, e^{-u} \frac{du}{j\beta\cos\alpha} = \frac{1}{j\beta}\int_0^0 e^{-u} du = 0$$

Using the above result will permit the evaluation of the integral:

$$\int_0^{2\pi} \sin\alpha\; e^{-j\beta\sin\alpha}\, d\alpha$$

$$\sin\alpha = -je^{j\alpha} + j\cos\alpha$$

$$\int_0^{2\pi} \sin\alpha\; e^{-j\beta\sin\alpha}\, d\alpha = -j\int_0^{2\pi} e^{j\alpha} e^{-j\beta\sin\alpha}\, d\alpha + j\int_0^{2\pi} \cos\alpha\; e^{-j\beta\sin\alpha}\, d\alpha$$

$$\int_0^{2\pi} \sin\alpha\; e^{-j\beta\sin\alpha}\, d\alpha = -j2\pi J_{-1}(\beta)$$

[14] I. S. Gradshteyn, I. M. Ryzhik, *Tables of Integrals Series and Products*, Academic Press, Inc., 1965.

$$J_{-1}(-\beta) = -J_1(-\beta) = J_1(\beta)$$

The J functions are Bessel functions of the first kind:

$$\int_0^{2\pi} \sin\alpha \; e^{-j\beta \sin\alpha} \, d\alpha = -j 2\pi J_1(\beta)$$

The next integral which must be evaluated is

$$\int_0^{2\pi} \cos^2\alpha \; e^{-j\beta \sin\alpha} \, d\alpha \; .$$

This integral can be worked by using the method of integration by parts:

$$u = \cos\alpha \qquad du = -\sin\alpha \, d\alpha$$

$$dv = \cos\alpha \; e^{-j\beta \sin\alpha} d\alpha \qquad v = \frac{j}{\beta} e^{-j\beta \sin\alpha}$$

$$\int_0^{2\pi} \cos^2\alpha \, e^{-j\beta \sin\alpha} \, d\alpha = \frac{j\cos\alpha}{\beta} \; e^{-j\beta \sin\alpha} \Big|_0^{2\pi} + \frac{j}{\beta}\int_0^{2\pi} \sin\alpha \; e^{-j\beta \sin\alpha} \, d\alpha$$

$$\int_0^{2\pi} \cos^2\alpha \; e^{-j\beta \sin\alpha} \, d\alpha = 0 + \frac{j}{\beta}\int_0^{2\pi} \sin\alpha \; e^{-j\beta \sin\alpha} \, d\alpha$$

$$\int_0^{2\pi} \cos^2\alpha \; e^{-j\beta \sin\alpha} \, d\alpha = \frac{2\pi \, J_1(\beta)}{\beta}$$

Another integral which needs to be evaluated:

$$\int_0^{2\pi} \sin^2\alpha \; e^{-j\beta \sin\alpha} \, d\alpha$$

$$\int_0^{2\pi} \sin^2\alpha \; e^{-j\beta \sin\alpha} \, d\alpha = \int_0^{2\pi} e^{-j\beta \sin\alpha} \, d\alpha - \int_0^{2\pi} \cos^2\alpha \; e^{-j\beta \sin\alpha} \, d\alpha$$

$$= 2\pi \left[J_0(\beta) - \frac{J_1(\beta)}{\beta} \right]$$

$$= 2\pi \left[\frac{J_1(\beta)}{\beta} - J_2(\beta) \right]$$

The last integral which needs to be evaluated is:

$$\int_0^{2\pi} \cos\alpha \; \sin\alpha \; e^{-j\beta\sin\alpha} \, d\alpha$$

This integral can be solved by using the method of integration by parts:

$u = \sin\alpha$ $\qquad$ $du = \cos\alpha \, d\alpha$

$dv = \cos\alpha \; e^{-j\beta\sin\alpha} \, d\alpha$ $\qquad$ $v = \frac{j}{\beta} e^{-j\beta\sin\alpha}$

$$\int_0^{2\pi} \sin\alpha \; \cos\alpha \; e^{-j\beta\sin\alpha} \, d\alpha = \frac{j\sin\alpha}{\beta} \; e^{-j\beta\sin\alpha} \Big|_0^{2\pi} + \frac{1}{j\beta}\int_0^{2\pi} \cos\alpha \; e^{-j\beta\sin\alpha} \, d\alpha$$

$$\int_0^{2\pi} \sin\alpha \; \cos\alpha \; e^{-j\beta\sin\alpha} \, d\alpha = 0$$